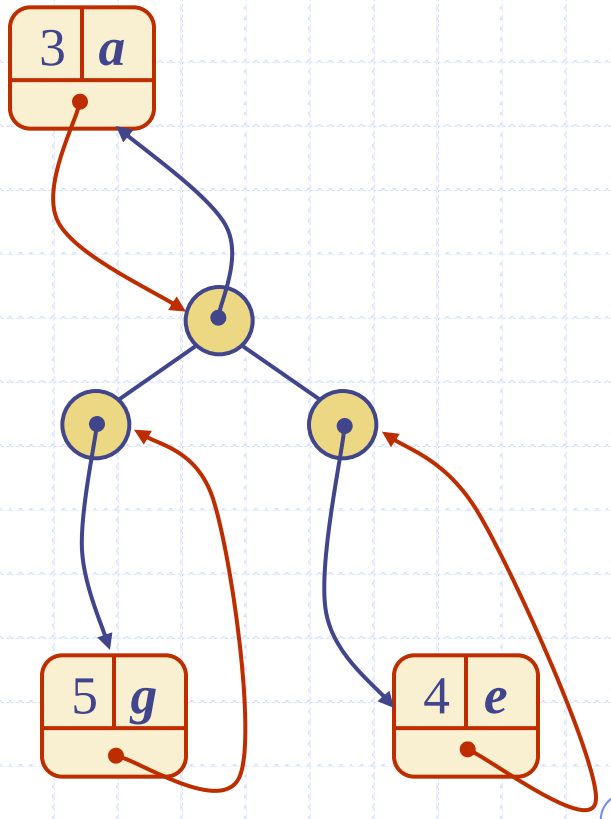
# Adaptable Priority Queues



# Entry and Priority Queue ADTs

- An entry stores a (key, value) pair
- Entry ADT methods:
  - getKey(): returns the key associated with this entry
  - getValue(): returns the value paired with the key associated with this entry
- Priority Queue ADT:
  - insert(k, x)
    inserts an entry with key k and value x
  - removeMin()
    removes and returns the entry with smallest key
  - min()
    returns, but does not remove, an entry with smallest key
  - size(), isEmpty()

# Example

- Online trading system where orders to purchase and sell a stock are stored in two priority queues (one for sell orders and one for buy orders) as (p,s) entries:
  - The key, p, of an order is the price
  - The value, s, for an entry is the number of shares
  - A buy order (p,s) is executed when a sell order (p′,s′) with price $p' \leq p$ is added (the execution is complete if $s' \geq s$)
  - A sell order (p,s) is executed when a buy order (p′,s′) with price $p' \geq p$ is added (the execution is complete if $s' \geq s$)
- What if someone wishes to cancel their order before it executes?
- What if someone wishes to update the price or number of shares for their order?

# Methods of the Adaptable Priority Queue ADT

- remove(e): Remove from P and return entry e.
- replaceKey(e,k): Replace with k and return the key of entry e of P; an error condition occurs if k is invalid (that is, k cannot be compared with other keys).
- replaceValue(e,x): Replace with x and return the value of entry e of P.

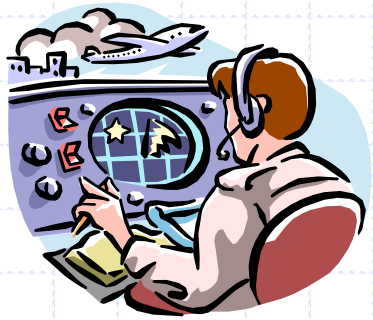
# Example

| *Operation* | *Output* | $P$ |
|---|---|---|
| insert$(5,A)$ | $e_1$ | $(5,A)$ |
| insert$(3,B)$ | $e_2$ | $(3,B),(5,A)$ |
| insert$(7,C)$ | $e_3$ | $(3,B),(5,A),(7,C)$ |
| min() | $e_2$ | $(3,B),(5,A),(7,C)$ |
| key$(e_2)$ | 3 | $(3,B),(5,A),(7,C)$ |
| remove$(e_1)$ | $e_1$ | $(3,B),(7,C)$ |
| replaceKey$(e_2,9)$ | 3 | $(7,C),(9,B)$ |
| replaceValue$(e_3,D)$ | $C$ | $(7,D),(9,B)$ |
| remove$(e_2)$ | $e_2$ | $(7,D)$ |

# Locating Entries

- In order to implement the operations remove(k), replaceKey(e), and replaceValue(k), we need fast ways of locating an entry e in a priority queue.
- We can always just search the entire data structure to find an entry e, but there are better ways for locating entries.

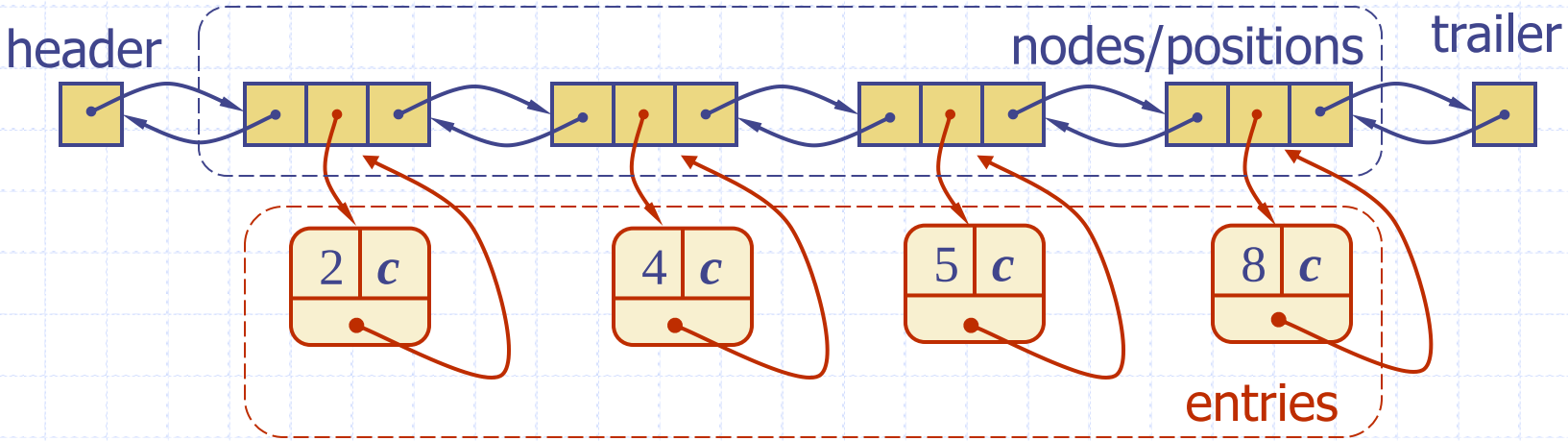
# Location-Aware Entries



- A locator-aware entry identifies and tracks the location of its (key, value) object within a data structure
- Intuitive notion:
  - Coat claim check
  - Valet claim ticket
  - Reservation number
- Main idea:
  - Since entries are created and returned from the data structure itself, it can return location-aware entries, thereby making future updates easier

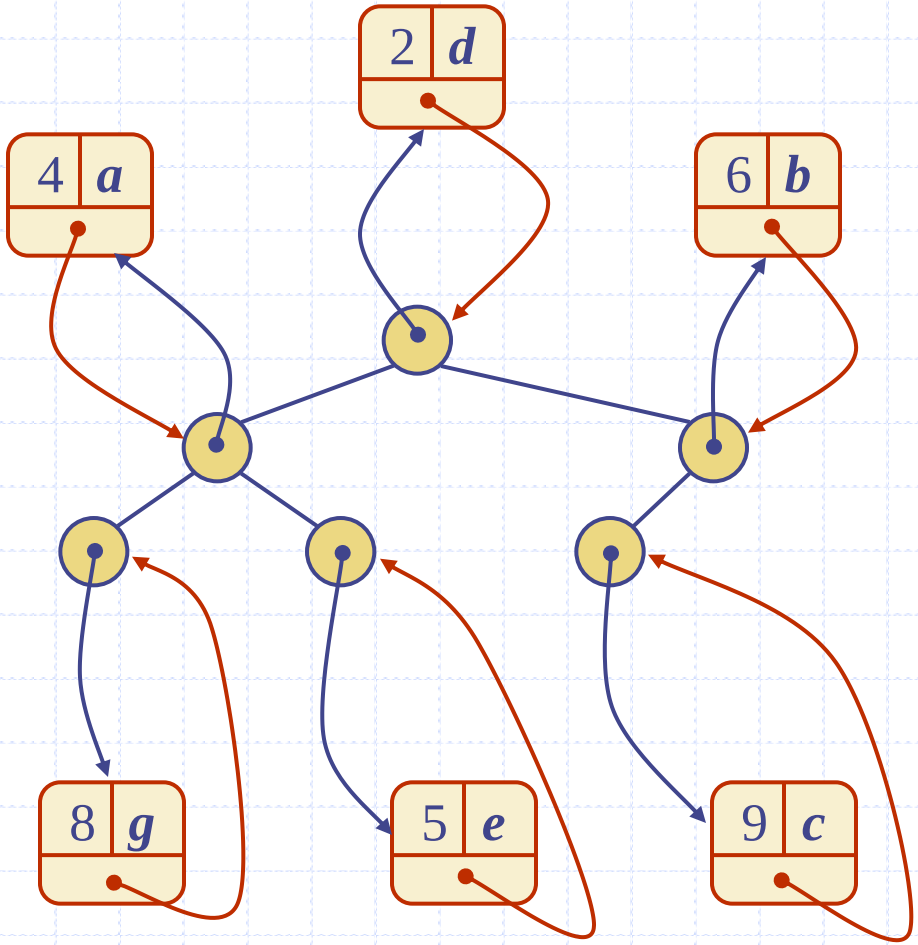
# List Implementation

- A location-aware list entry is an object storing
  - key
  - value
  - position (or rank) of the item in the list
- In turn, the position (or array cell) stores the entry
- Back pointers (or ranks) are updated during swaps



# Heap Implementation

- A location-aware heap entry is an object storing
  - key
  - value
  - position of the entry in the underlying heap
- In turn, each heap position stores an entry
- Back pointers are updated during entry swaps



# Performance

- Improved times thanks to location-aware entries are highlighted in red

| Method | Unsorted List | Sorted List | Heap |
|---|---|---|---|
| size, isEmpty | $O(1)$ | $O(1)$ | $O(1)$ |
| insert | $O(1)$ | $O(n)$ | $O(\log n)$ |
| min | $O(n)$ | $O(1)$ | $O(1)$ |
| removeMin | $O(n)$ | $O(1)$ | $O(\log n)$ |
| remove | $\boldsymbol{O(1)}$ | $\boldsymbol{O(1)}$ | $\boldsymbol{O(\log n)}$ |
| replaceKey | $\boldsymbol{O(1)}$ | $O(n)$ | $\boldsymbol{O(\log n)}$ |
| replaceValue | $\boldsymbol{O(1)}$ | $\boldsymbol{O(1)}$ | $\boldsymbol{O(1)}$ |