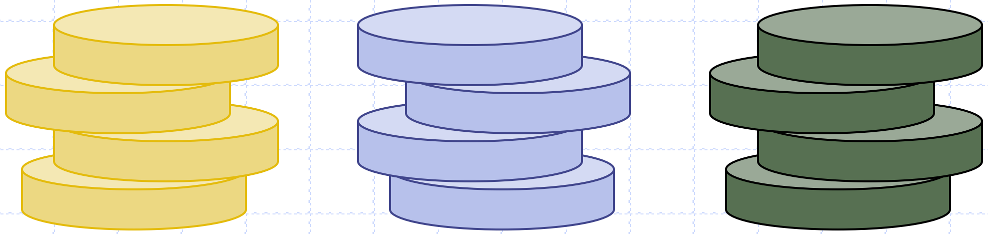
# Stacks



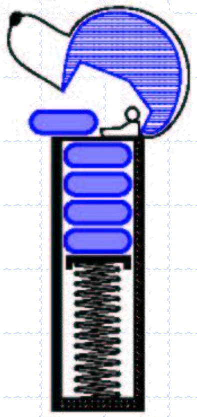
## Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations

- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

# The Stack ADT

- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored



# Stack Interface in Java

- Java interface corresponding to our Stack ADT
- Requires the definition of class EmptyStackException
- Different from the built-in Java class java.util.Stack

```
public interface Stack<E> {

  public int size();

  public boolean isEmpty();

  public E top()
        throws EmptyStackException;

  public void push(E element);

  public E pop()
        throws EmptyStackException;
}
```

## Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an exception
- Exceptions are said to be “thrown” by an operation that cannot be executed
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty
- Attempting the execution of pop or top on an empty stack throws an EmptyStackException

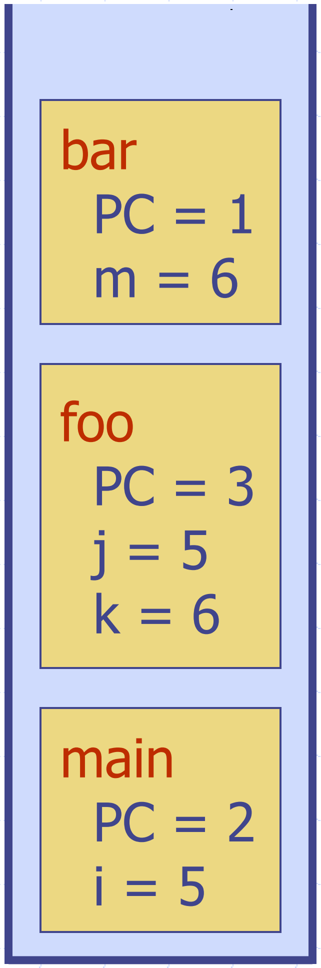
## Applications of Stacks

- Direct applications
    - Page-visited history in a Web browser
    - Undo sequence in a text editor
    - Chain of method calls in the Java Virtual Machine
- Indirect applications
    - Auxiliary data structure for algorithms
    - Component of other data structures

# Method Stack in the JVM

- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack
- When a method is called, the JVM pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack
- Allows for recursion

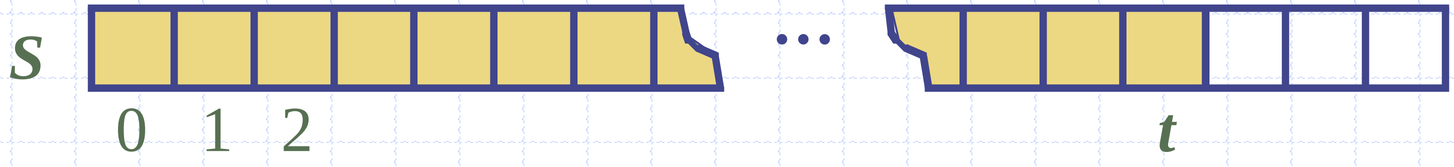
```
main() {
   int i = 5;
   foo(i);
}

foo(int j) {
   int k;
   k = j+1;
   bar(k);
}

bar(int m) {
   …
}
```



# Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

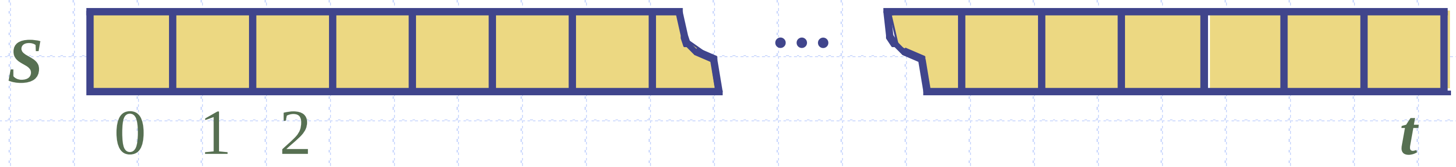
```
Algorithm size()
    return $t + 1$

Algorithm pop()
    if isEmpty() then
        throw EmptyStackException
    else
        $t \leftarrow t - 1$
        return $S[t + 1]$
```



## Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a FullStackException
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

```
Algorithm push(o)
  if $t = S.length - 1$ then
    throw FullStackException
  else
    $t \leftarrow t + 1$
    $S[t] \leftarrow o$
```



## Performance and Limitations

- Performance
  - Let $n$ be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

## Array-based Stack in Java

```
public class ArrayStack<E>
      implements Stack<E> {

   // holds the stack elements
   private E S[ ];

   // index to top element
   private int top = -1;

   // constructor
   public ArrayStack(int capacity) {
       S = (E[]) new Object[capacity]);
   }
```

```
    public E pop()
          throws EmptyStackException {
      if isEmpty()
        throw new EmptyStackException
              ("Empty stack: cannot pop");
        E temp = S[top];
        // facilitate garbage collection:
        S[top] = null;
        top = top – 1;
        return temp;
     }
…  (other methods of Stack interface)
```

# Example use in Java

```
public class Tester {

    // … other methods
    public intReverse(Integer a[]) {
        Stack<Integer> s;
        s = new ArrayStack<Integer>();

        … (code to reverse array a) …
    }
```

```
public floatReverse(Float f[]) {
    Stack<Float> s;
    s = new ArrayStack<Float>();

    … (code to reverse array f) …
}
```

# Parentheses Matching

- Each "(", "{", or "[" must be paired with a matching ")", "}", or "["
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}
  - incorrect: )(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

# Parentheses Matching Algorithm

```
Algorithm ParenMatch(X,n):
Input: An array X of n tokens, each of which is either a grouping symbol, a
variable, an arithmetic operator, or a number
Output: true if and only if all the grouping symbols in X match
Let S be an empty stack
for i=0 to n-1 do
    if X[i] is an opening grouping symbol then
        S.push(X[i])
    else if X[i] is a closing grouping symbol then
        if S.isEmpty() then
            return false {nothing to match with}
        if S.pop() does not match the type of X[i] then
            return false {wrong type}
if S.isEmpty() then
    return true {every symbol matched}
else return false {some symbols were never matched}
```

## HTML Tag Matching

- For fully-correct HTML, each <name> should pair with a matching </name>

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```

The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

## Tag Matching Algorithm (in Java)

```
import java.io.*;
import java.util.Scanner;
import net.datastructures.*;
/** Simplified test of matching tags in an HTML document. */
public class HTML {
  /** Strip the first and last characters off a <tag> string. */
  public static String stripEnds(String t) {
    if (t.length() <= 2) return null;            // this is a degenerate tag
    return t.substring(1,t.length()-1);
  }
  /** Test if a stripped tag string is empty or a true opening tag. */
  public static boolean isOpeningTag(String tag) {
    return (tag.length() == 0) || (tag.charAt(0) != '/');
  }
```

# Tag Matching Algorithm (cont.)

```
/** Test if stripped tag1 matches closing tag2 (first character is '/'). */
 public static boolean areMatchingTags(String tag1, String tag2) {
   return tag1.equals(tag2.substring(1)); // test against name after '/'
 }
 /** Test if every opening tag has a matching closing tag. */
 public static boolean isHTMLMatched(String[] tag) {
   Stack<String> S = new NodeStack<String>();     // Stack for matching tags
   for (int i = 0; (i < tag.length) && (tag[i] != null); i++) {
     if (isOpeningTag(tag[i]))
     S.push(tag[i]); // opening tag; push it on the stack
     else {
     if (S.isEmpty())
       return false;                        // nothing to match
     if (!areMatchingTags(S.pop(), tag[i]))
       return false;      // wrong match
     }
     }
   if (S.isEmpty()) return true; // we matched everything
   return false; // we have some tags that never were matched
 }
```

# Tag Matching Algorithm (cont.)

```
public final static int CAPACITY = 1000; // Tag array size
/* Parse an HTML document into an array of html tags */
public static String[] parseHTML(Scanner s) {
  String[] tag = new String[CAPACITY]; // our tag array (initially all null)
  int count = 0;                       // tag counter
  String token;                        // token returned by the scanner s
  while (s.hasNextLine()) {
    while ((token = s.findInLine("<[^>]*>")) != null)  // find the next tag
      tag[count++] = stripEnds(token); // strip the ends off this tag
    s.nextLine(); // go to the next line
  }
  return tag; // our array of (stripped) tags
}
public static void main(String[] args) throws IOException { // tester
  if (isHTMLMatched(parseHTML(new Scanner(System.in))))
    System.out.println("The input file is a matched HTML document.");
  else
    System.out.println("The input file is not a matched HTML document.");
}
}
```

## Evaluating Arithmetic Expressions

Slide by Matt Stallmann included with permission.

14 – 3 * 2 + 7 = (14 – (3 * 2) ) + 7

Operator precedence

* has precedence over +/–

Associativity

operators of the same precedence group evaluated from left to right
Example: (x – y) + z rather than x – (y + z)

Idea: push each operator on the stack, but first pop and perform higher and *equal* precedence operations.

# Algorithm for Evaluating Expressions

Slide by Matt Stallmann included with permission.

Two stacks:

- opStk holds operators
- valStk holds values
- Use $ as special “end of input” token with lowest precedence

Algorithm doOp()

```
x ← valStk.pop();
y ← valStk.pop();
**op** ← opStk.pop();
valStk.push( y **op** x )
```

Algorithm repeatOps( refOp ):

```
 while ( valStk.size() > 1 ∧
          prec(refOp) ≤
          prec(opStk.top())
   doOp()
```

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

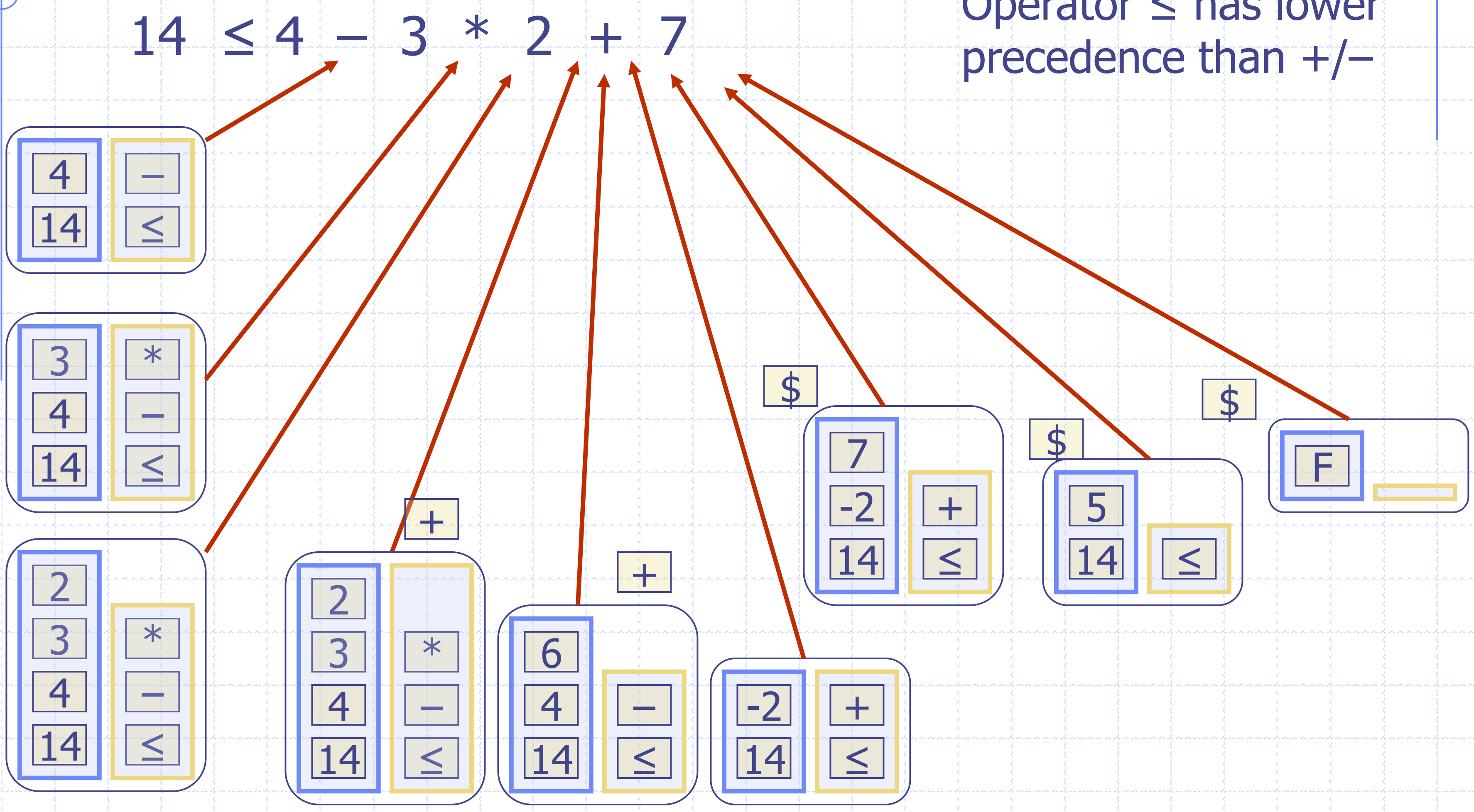
Output: the value of the expression

```
while there’s another token z
    if isNumber(z) then
        valStk.push(z)
    else
        repeatOps(z);
        opStk.push(z)
repeatOps($);
return valStk.top()
```

# Algorithm on an Example Expression

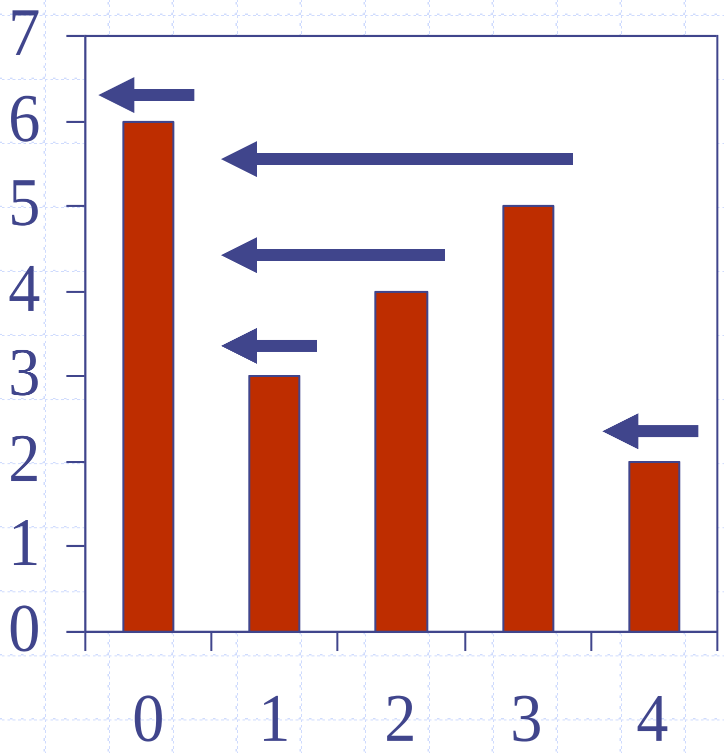
Slide by Matt Stallmann included with permission.

Operator ≤ has lower precedence than +/–



# Computing Spans (not in book)

- Using a stack as an auxiliary data structure in an algorithm
- Given an an array $X$, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ and such that $X[j] \leq X[i]$
- Spans have applications to financial analysis
  - E.g., stock at 52-week high



|  |  |  |  |  |  |
|---|---|---|---|---|---|
| $X$ | 6 | 3 | 4 | 5 | 2 |
| $S$ | 1 | 1 | 2 | 3 | 1 |

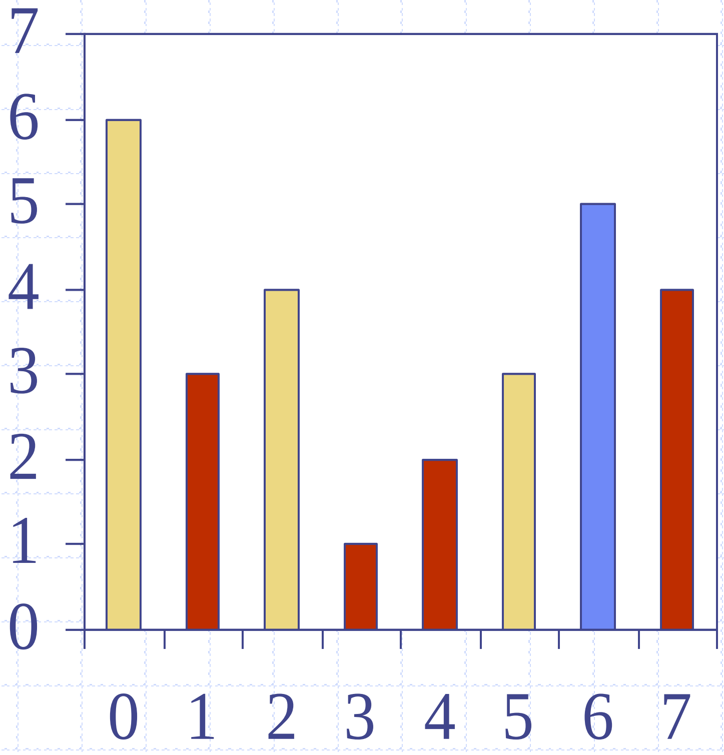
# Quadratic Algorithm

```
Algorithm spans1(X, n)
    Input array X of n integers
    Output array S of spans of X                    #
    S ← new array of n integers                     $n$
    for $i ← 0$ to $n - 1$ do                         $n$
        $s ← 1$                                     $n$
        while $s \le i \wedge X[i - s] \le X[i]$     $1 + 2 + \ldots + (n - 1)$
            $s ← s + 1$                             $1 + 2 + \ldots + (n - 1)$
        $S[i] ← s$                                  $n$
    return S                                        1
```

- Algorithm ***spans1*** runs in $O(n^2)$ time

# Computing Spans with a Stack

- We keep in a stack the indices of the elements visible when "looking back"
- We scan the array from left to right
  - Let $i$ be the current index
  - We pop indices from the stack until we find index $j$ such that $X[i] < X[j]$
  - We set $S[i] \leftarrow i - j$
  - We push $x$ onto the stack



## Linear Algorithm

- Each index of the array
  - Is pushed into the stack exactly one
  - Is popped from the stack at most once
- The statements in the while-loop are executed at most $n$ times
- Algorithm *spans2* runs in $O(n)$ time

```
Algorithm spans2(X, n)                          #
    S ← new array of n integers                 n
    A ← new empty stack                         1
    for i ← 0 to n − 1 do                       n
        while (¬A.isEmpty() ∧
                X[A.top()] ≤ X[i] ) do          n
            A.pop()                             n
        if A.isEmpty() then                     n
            S[i] ← i + 1                        n
        else
            S[i] ← i − A.top()                  n
        A.push(i)                               n
    return S                                    1
```