Abstract: Modular curves and their compactifications are of fundamental importance in number theory. A key property of modular curves is that they are moduli spaces: their points classify certain geometric objects (elliptic curves equipped with level structure). Similarly, it was shown by Deligne-Rapoport that compactified modular curves may be viewed as moduli spaces for "generalized" elliptic curves equipped with level structure.

It was shown by Abramovich-Olsson-Vistoli that modular curves naturally lie inside certain complicated moduli spaces, classifying "twisted stable maps" to certain algebraic stacks. These moduli spaces turn out to be complete, so the closure of a modular curve inside such a moduli space gives a compactification of the modular curve. In this talk I explain how these new compactifications can themselves be viewed as moduli spaces, and I compare them to the "classical" compactified modular curves considered by Deligne-Rapoport.