A reexamination of the Birch and Swinnerton-Dyer cubic surfaces

Mckenzie West
Emory University

Abstract: The Hasse principle asks whether solutions to an equation in a local field extend to those in a global field. This does not always happen, the Brauer-Manin obstruction being a common explanation. A conjecture of Colliot-Thélène and Sansuc implies that a Brauer-Manin obstruction exists for every cubic surface which fails to satisfy the Hasse principle. In 1975, Birch and Swinnerton-Dyer gave some early examples of cubic surfaces which have a Brauer-Manin obstruction:

\[(\text{cubic norm}) = (\text{linear}) (\text{quadratic norm}).\]

They make a rough number theoretic argument for the Brauer-Manin obstruction in the case that the Hasse principle fails, focusing on the particular fields and constants. We make use of advancements in arithmetic geometry, taking a geometric look at these objects and utilizing the correspondence between the Brauer group and the Picard group of a surface in order to update and generalize their arguments.

Tuesday, November 10, 2015, 4:00 pm
Mathematics and Science Center: W304