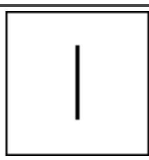
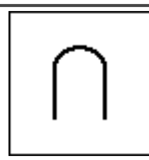
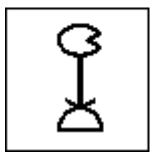
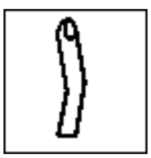
The Egyptian Number System

The number system of ancient Egypt was decimal in nature but did not make use of place value. As a result of this, they didn't have a symbol for zero. There were two kind of writing used in Egypt, hieroglyphics and demotic (hieratic). In the hieroglyphic system, the symbols used were:

1	vertical stroke
10	heel bone
100	coiled rope (snare)
1,000	lotus flower
10,000	bent finger
100,000	burbot fish
1,000,000	kneeling figure

 =1  =10  =100

 =1,000

 =10,000  =100,000  =1,000,000

The number one thousand, three hundred forty two (1,342) would look like :

In many cases, hieroglyphs were stacked rather than written in a single straight line.

Rosetta Stone

The Ancient Egyptians used both several methods of writing , including hieroglyphic, hieratic, and demotic scripts. The story behind how "modern" scholars were able to decipher these methods of writing is the story of the Rosetta Stone (link courtesy of the British Museum where the Rosetta Stone is housed). Hieroglyphic script was used by Egyptians for important or religious documents, while the demotic script was a simplified version of hieroglypics, and was the writing method for "the common". Demotic script evolved from hierotic, and was used during the "last period" of ancient Egyptian, a 1000 year span from 500 BC to 500 AD. By 400 AD, demotic script was replaced almost entirely by the use of Greek writing.

The Rosetta Stone, carved in 200BC, was written in three scripts (hieroglyphic, demotic, and Greek) so that the priests, government officials and rulers of Egypt could read what it said. It was found by Napolean's soldiers in 1800 (1799), during the French occupation of Egypt. After the British defeated Naplolean, the Rosetta Stone was moved to Britain, where it now resides in the British Museum

Egyptian Fractions and the Rhind Papyrus

The Ancient Egyptians used unit fractions e.g. e.g 1/4, 1/7, 1/15. Unit fractions are those positive rational numbers (fractions) which have the number 1 as numerator . Their usual way of writing fractions was to use the word r, meaning part, with the denominator written below and, if need be, beside it as well e.g.

="part 12" which is equivalent to our twelfth or 1/12.

Non unit-fractions like 2/5 or 7/8 did not exist- though there was one exception : 2/3.

Unit fractions were used because we can represent any fraction by adding unit fractions together e.g. 4/7 would be written as 1/2

+ 1/14. (see class notes for a the explanation of how a unit fraction decomposition is obtained)

To represent the sum of 1/3 and 1/5 for example, they would simply write 1/3 + 1/5, wheras we might represent this as a single fraction 8/15 . The problem the Egyptians had was that although they had a notation for the unit fractions, i.e., fractions of the form 1/n , they did not have a compact notation for the general fraction m/n . Some might say their numeration system was faulty, but that would be overly critical, since they were the first (as far as we know) to have *any* way of giving names to fractions.

Question : How would an Egyptian scribe would have written 3/8 ? 3/5 ?

> Answer:One can write 3/8 = 1/2 + 1/8, and 3/5 = 1/2 + 1/10

Even though the Egyptian method of writing fractions continued to be used for a long time, there were many limitations to it s use. In the work the **Almagast** , written by the Greek scientist Ptolemy in the first century AD, Ptolemy uses the ancient Bablonian method of writing fractions (sexagesimal) rather than the Egyptian method because of the embarrassments that the Egyptian method often cause.

Much of what is known today about Egyptian fractions has been deduced from the**Rhind papyrus** , written by the scribe Ahmes around 1650 BC. This book consists mainly of 84 word problems of a diverse nature, plus a few tables to aid the young scriblets (young scribes) that Ahmes taught the art of calculation. Generally speaking, Ahmes seemed to be happy to write the answer to a problem as a whole number plus a sum of unit fractions, with no unit fraction appearing more than once in an answer.

Problem Establish the following algebraic identities

- (*) For any n except 0, 1/n = 1/(n+1) + 1/n(n+1)
- (**) If n is odd, 2/n = 2/(n+1) + 2/n(n+1) = a sum of unit fractions, since 2/(n+1) and 2/n(n+1) can be reduced.
- (***) 2/n = 1/n + 1/(n+1) + 1/n(n+1)

- (****)   1 = 1/2 + 1/3 + 1/6

Solution:. (easy)

With these identities, once can show that there is no unique unit fraction decomposition of a given fraction.

Tables in the Rhind Papyrus

The largest table in the Rhind Papyrus is the 2/n table, where Ahmes gives decompositions of these fractions into sums of unit fractions. Most of the entries in the table come from the second identity (**) which is obtained by multiplying (*) by 2 on both sides . Thus

2/7 = 2/8 + 2/(7*8) = 1/4 + 1/28

Note that using (***) we obtain:

2/7 = 1/7 + 1/8 + 1/56

an expression which is "longer".

Fibonnaci's theorem

The Egyptian system of writing fractions as sums of unit fractions continued in use even after much more efficient systems were developed. Fibonnaci was aware of the system in 1200 AD and included in his book **Liber Abaci** a method for writing any fraction as a sum of unit fractions. His method was one which might be the most "natural":

> **Fibonnaci's method** : Take the fraction you wish to express in the Egyptian manner and subtract from it the largest unit fraction which is not larger than it. If the remainder is not itself a unit fraction, then repeat the process on the remainder. Continue this until the remainder is a unit fraction.

For example, if the fraction is 4/5, we note that the largest unit fraction not larger than 4/5 is 1/2 (5 DIV 4 + 1 give the value of the denominator of such a unit fraction) . Subtract 1/2 to obtain 3/10. The largest unit fraction not larger than 3/10 is 1/4. Subtract 1/4 to obtain 1/20.

Hence, Fibonnaci's method leads to the decomposition

$$4/5 = 1/2 + 1/4 + 1/20$$

There is no record indicating that Fibonnaci had a "proof" that his method always worked, but a proof that it does always work can be based on the following lemma.

Lemma Let p/q be any fraction which is not a unit fraction. Let $1/n$ be the largest unit fraction less than or equal to p/q . Then $p/q - 1/n$ is a fraction r/s, with $r < p$. .
Proof (see class notes)

Theorem Fibbonaci's method works for all fractions p/q
Proof (see class notes)

Online Resources for UNIT FRACTIONS

- Egyptian Fractions (Math 3031 website)
- R. Knott's Egyptian Math site (great site)