Subcontracting

One of the ways to deal with temporary overload is subcontracting. When the total number of jobs in the system becomes too high, the production lead time (waiting time plus processing time) will be too high as well. In order to stay away from this situation one might use subcontracting. Some time before the actual overload occurs, some of the jobs are sent to subcontracters. So these jobs will not enter the production system, but will be executed elsewhere. It is important to know how this influences the mean production lead time and how many jobs are not accepted.

Let us look at the following simple system. Jobs arrive at a single machine according to a Poisson process with rate λ. The arrival stream consists of two classes of jobs, 1 and 2. The jobs from both classes have exponential processing times with the same mean $1/\mu$. With α we denote the probability that an arriving job is of class 1.

Class 1 jobs have to be accepted, whereas class 2 jobs can be sent to a subcontractor. We assume that the decision whether or not to accept a job is only taken upon arrival. The natural subcontracting rule is: accept all jobs as long as the number of jobs in the system is less than N and do not accept class 2 jobs whenever the system contains N or more jobs. Accepted jobs are served in order of arrival, so FCFS.

This system can be described by a Markov process. The state can be characterized by the total number of jobs in the system.

(i) Explain why this simple state description is sufficient.

Let p_k be the equilibrium probability of having k jobs in the system, $k = 0, 1, 2, \ldots$.

(ii) Formulate and solve the balance equations for the probabilities p_k.

Important performance measures for this model are P_{rej}, the fraction of the jobs from class 2 that is not accepted, and $E(S_1)$ and $E(S_2)$, the mean production lead times for the jobs of the two classes.

(iii) Use the PASTA property to determine P_{rej}, $E(S_1)$ and $E(S_2)$.

Note that for class 2 jobs we have to be careful. The mean production lead time for an arbitrary class 2 job is not the same as the mean production lead time for an *accepted* class 2 job. Let us denote the latter by $E(S_2|\text{accepted})$.

(iv) Give the relation between $E(S_2)$ and $E(S_2|\text{accepted})$.

(v) Indicate how the distribution of the production lead time of an arriving job, that enters the system, can be determined.

Let us now consider two situations, one with a basic load of 0.95 and the other with a load of 1.05. The latter system would explode without a subcontracting rule. Both classes contribute 50 percent to the load, so by not accepting class 2 jobs the load drops to 0.475 and 0.525, respectively.

(vi) Complete the following table (take $\mu = 1$ in each example):

ρ	N	$E(S_1)$	$E(S_2)$	P_{rej}
0.95	∞			
	40			
	20			
	10			
1.05	∞			
	40			
	20			
	10			

(vii) What are your conclusions from the results in the table above?