Unreliable machines

We consider a single-stage production system consisting of two identical machines working in parallel. Jobs arrive according to a Poisson stream with rate λ and the processing times are exponential with mean $1/\mu$. The processing discipline is First Come First Served. The machines, however, are not perfectly reliable, but they are subject to operational failures. That is, machines can only fail while they are processing jobs. The time to failure is exponentially distributed with mean $1/\eta$. There is one dedicated repair person. So if one machine is undergoing repair when the other fails, there will be a delay before repair of the second machine can begin. The repair time is exponentially distributed with mean $1/\theta$.

(i) Determine ρ_i, the fraction of time i ($i = 0, 1, 2$) machines are operational, given that there are always jobs waiting for being processed.

(ii) Show that the production system is stable (i.e., the number of jobs in the system does not grow to infinity) if and only if

$$\lambda < \rho_1 \cdot \mu + \rho_2 \cdot 2\mu.$$

The production system can be described by a two-dimensional Markov process with states (i, j), $i = 0, 1, 2, \ldots$, $j = 0, 1, 2$, where i is the total number of jobs in the system and j the number of machines that is operational.

(iii) Formulate and solve the balance equations for the equilibrium probabilities $p(i, j)$.

(iv) Determine the following performance characteristics:

a. The mean waiting time of a job, $E(W)$;

b. The mean production lead time of a job, $E(S)$ (i.e., the mean time that elapses from the arrival of a job till the time the job is finished);

c. The mean number of machines that is waiting for repair, $E(L_M^q)$, and the mean time a machine is waiting for repair, $E(W_M)$;

d. The utilization rate of the repair person, ρ_R.

(v) Suppose that $\mu = 1$ and $\eta/\theta = 0.1$. Complete the following table:

λ	η	$E(W)$	$E(S)$	$E(L_M^q)$
1.5	1			
	0.025			
1.7	1			
	0.025			