

Waiting time analysis of a $U/PH/1$ system

Consider a machine processing jobs. The mean processing time is 4 minutes and the standard deviation is 3 minutes. The mean number of jobs arriving per hour is 10. The interarrival times are uniformly distributed between 4 and 8 minutes.

- (i) Show that, if the processing time is modelled as a mixture of an Erlang-1 (i.e. exponential) distribution and an Erlang-2 distribution with density

$$f_B(t) = p\mu e^{-\mu t} + (1-p)\mu^2 t e^{-\mu t},$$

the parameters p and μ can be chosen such that this distribution fits the mean and standard deviation of the processing time.

The state of this system just prior to the arrival of a job can be described by n , where n is the total number of uncompleted processing phases in the system.

- (ii) Formulate and solve the balance equations for the arrival probabilities p_n .
- (iii) Determine the mean waiting time of a job.
- (iv) What is the fraction of jobs that has to wait longer than 10 minutes?