Subcontracting

(i) In this state description information about the class of the jobs is neglected. But it is irrelevant which classes the jobs in the system belong to, since both classes of jobs have the same (exponential) processing times.

(ii) By balancing the flow between state k and $k+1$ we get

$$\begin{aligned} \lambda p_k &= \mu p_{k+1}, \qquad 0 \le k < N \\ \alpha\lambda p_k &= \mu p_{k+1}, \qquad k \ge N. \end{aligned}$$

So, with $\rho = \lambda/\mu$,

$$p_k = \rho^k p_0, \qquad 0 \le k < N,$$

nd

$$p_{N+k} = \rho^N (\alpha\rho)^k p_0, \qquad k \ge 0.$$

From the normalization, $\sum_0^\infty p_k = 1$, we get

$$\frac{1}{p_0} = \sum_{k=0}^{N-1} \rho^k + \frac{\rho^N}{1-\alpha\rho} = \frac{1-\rho^N}{1-\rho} + \frac{\rho^N}{1-\alpha\rho}.$$

(iii) Using PASTA we have

$$\begin{aligned} P_{rej} &= \sum_{k=0}^{\infty} p_{N+k} = p_0 \rho^N/(1-\alpha\rho), \\ E(S_2) &= p_0 \sum_{k=0}^{N-1} \rho^k (k+1) \frac{1}{\mu} \\ &= p_0 \frac{1-\rho^{N+1} - (N+1)\rho^N(1-\rho)}{(1-\rho)^2} \frac{1}{\mu}, \end{aligned}$$

and

$$\begin{aligned} E(S_1) &= p_0 \sum_{k=0}^{N-1} \rho^k (k+1) \frac{1}{\mu} + p_0 \sum_{k=0}^{\infty} (\alpha\rho)^k \rho^N (k+N+1) \frac{1}{\mu} \\ &= p_0 \left(\frac{1-\rho^{N+1} - (N+1)\rho^N(1-\rho)}{(1-\rho)^2} + \frac{\rho^N N}{1-\alpha\rho} + \frac{\rho^N}{(1-\alpha\rho)^2} \right) \frac{1}{\mu}. \end{aligned}$$

(iv) $E(S_2) = P_{rej} \cdot 0 + (1 - P_{rej}) \cdot E(S_2|\text{accepted}).$

ρ	N	$E(S_1)$	$E(S_2)$	P_{rej}
0.95	∞	20	20	0.000
	40	14.5	13.9	0.014
	20	9.5	8.4	0.051
	10	5.9	4.4	0.124
1.05	∞	∞	∞	0.000
	40	28.3	23.7	0.109
	20	13.5	10.3	0.145
	10	7.2	4.6	0.214

Table 1: Performance under subcontracting.

(v) Condition on the number of jobs present when an arriving job enters the system. If there are k jobs present, then the production lead time of the arriving job is Erlang distributed with parameters $k+1$ and μ.

(vi) Table 1 shows results for $\alpha = 0.5$, $1/\mu = 1$ and $\rho = 0.95$ and $\rho = 1.05$, respectively.

(vii) From the results in table 1 we see that by sending a limited amount of jobs to subcontractors, the performance improves considerably. For $N = 20$ and $\rho = 0.95$ only 5 percent of the jobs from stream 2 are sent to subcontractors while the throughput time is reduced by more than 50 percent. What we further see is that the system that would explode without a subcontracting or rejection option behaves quite well for $N = 20$ or $N = 10$.

Points

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
1	2	2	1	1	2	1