

Loading of containers

- (i) The system is stable if (and only if)

$$\lambda \cdot (E(B_1) + E(B_2)) < 1.$$

- (ii) Let $\rho_i = \lambda E(B_i)$ and $E(R_i) = E(B_i^2)/2E(B_i)$ for $i = 1, 2$. Then the mean waiting time is given by

$$E(W) = \rho_1(E(R_1) + E(B_2)) + \rho_2 E(R_2) + E(L^q)(E(B_1) + E(B_2)),$$

and by Little's law we have

$$E(L^q) = \lambda E(W).$$

Hence we get

$$E(W) = \frac{\rho_1(E(R_1) + E(B_2)) + \rho_2 E(R_2)}{1 - \rho_1 - \rho_2}.$$

Since $\lambda = 1/15$ slaves per minute, $E(B_1) = 3$, $\sigma(B_1) = 2$, $E(B_2) = 10$ and $\sigma(B_2) = 8$ (so $E(R_1) = 13/6$ and $E(R_2) = 41/5$), we find

$$E(W) = \frac{237}{4} = 59\frac{1}{4} \text{ min.}$$

The mean sojourn time follows by adding the mean fetch time, so

$$E(S) = E(W) + E(B_1) = 62\frac{1}{4} \text{ min.}$$

- (iii) The system is stable if (and only if)

$$\lambda \cdot (E(C_1) + E(C_2)) < 1.$$

- (iv) Now let $\rho_i = \lambda E(C_i)$ and $E(R_i) = E(C_i^2)/2E(C_i)$ for $i = 1, 2$. For the mean sojourn time we have

$$\begin{aligned} E(S) &= (E(L) + 1)(E(C_1) + E(C_2)) \\ &\quad + (1 - \rho_1 - \rho_2)(E(C_2) - (E(C_1) + E(C_2))) \\ &\quad + \rho_1(E(R_1) + E(C_2) - (E(C_1) + E(C_2))) \\ &\quad + \rho_2(E(R_2) - (E(C_1) + E(C_2))). \end{aligned}$$

The first term at the right-hand side states that the mean (residual) service time of all slaves in the queue is $E(C_1) + E(C_2)$. This is obviously not correct; the other three terms are corrections. For example, if on arrival the crane is idle (with

probability $1 - \rho_1 - \rho_2$) then the service time of the arriving slave is $E(C_2)$ instead of $E(C_1) + E(C_2)$. Together with Little's law, i.e. $E(L) = \lambda E(S)$, we find

$$E(S) = \frac{\rho_1(E(R_1) + E(C_2)) + \rho_2 E(R_2)}{1 - \rho_1 - \rho_2} + E(C_2) = 62\frac{1}{4} \text{ min.}$$

If we define the waiting time as the time elapsing from the arrival of a slave until the crane starts loading the container on that slave, then $S = W + C_2$ and thus

$$E(W) = E(S) - E(C_2) = 59\frac{1}{4} \text{ min.}$$

Hence, we obtain the same results as in (ii). This is no surprise by observing that the sojourn time remains the same when we act as if the service time of a slave is C_2 followed by C_1 (instead of the other way around), and if the slave may leave as soon as C_2 has been finished.

Points

(i)	(ii)	(iii)	(iv)
1	4	1	4