Priorities

Let λ_i the arrival rate of type i jobs, $E(B_i)$ the mean processing time, $E(R_i)$ the mean residual processing time and ρ_i the occupation rate for type i jobs, $i = 1, 2, 3$. Then we have $\lambda_1 = 2$ jobs per hour, $E(B_1) = 1/6$ hour, $E(R_1) = E(B_1)/2 = 1/12$ hour and $\rho_1 = \lambda_1 E(B_1) = 1/3$. Similarly we find $\lambda_2 = \lambda_3 = 1$ job per hour, $E(B_2) = 1/4$ hour, $E(B_3) = 1/3$ hour, $E(R_2) = 1/8$ hour, $E(R_3) = 1/6$ hour, $\rho_2 = 1/4$ and $\rho_3 = 1/3$. Further, let λ denote the total arrival rate, $E(B)$ the mean processing time of an arbitrary job and ρ the total utilization rate; so $\lambda = 4$ jobs per hour, $E(B) = E(B_1)/2 + E(B_2)/4 + E(B_3)/4 = 22/96$ hour and $\rho = \lambda E(B) = 11/12$.

(i) Let $E(W)$ denote the mean waiting time and $E(L^q)$ the mean number of jobs waiting in the queue. Then

$$E(W) = E(L^q)E(B) + \rho_1 E(R_1) + \rho_2 E(R_2) + \rho_3 E(R_3)$$

and

$$E(L^q) = \lambda E(W).$$

Hence we obtain

$$E(W) = \frac{\rho_1 E(R_1) + \rho_2 E(R_2) + \rho_3 E(R_3)}{1 - \rho} = \frac{11}{8} \text{ hour} = 82.5 \text{ min.}$$

For the mean sojourn time of type i jobs we have

$$E(S_i) = E(W) + E(B_i), \qquad i = 1, 2, 3,$$

so

$$\begin{aligned} E(S_1) &= 1\frac{13}{24} = 1.54 \text{ hour} = 92.5 \text{ min}, \\ E(S_2) &= 1\frac{5}{8} = 1.63 \text{ hour} = 97.5 \text{ min}, \\ E(S_3) &= 1\frac{17}{24} = 1.71 \text{ hour} = 102.5 \text{ min}. \end{aligned}$$

The overall mean sojourn time is given by

$$E(S) = \frac{\lambda_1}{\lambda} E(S_1) + \frac{\lambda_2}{\lambda} E(S_2) + \frac{\lambda_3}{\lambda} E(S_3) = 1.60 \text{ hour} = 96.25 \text{ min.}$$

(ii) The mean sjourn times are now given by

$$\begin{aligned} E(S_1) &= \frac{\rho_1 E(R_1) + \rho_2 E(R_2) + \rho_3 E(R_3)}{1 - \rho_1} + E(B_1) = \frac{65}{192} = 0.34 \text{ hour} = 20.3 \text{ min}, \\ E(S_2) &= \frac{\rho_1 E(R_1) + \rho_2 E(R_2) + \rho_3 E(R_3)}{(1 - \rho_1 - \rho_2)(1 - \rho_1)} + E(B_2) = \frac{53}{80} = 0.66 \text{ hour} = 39.8 \text{ min}, \\ E(S_3) &= \frac{\rho_1 E(R_1) + \rho_2 E(R_2) + \rho_3 E(R_3)}{(1 - \rho_1 - \rho_2 - \rho_3)(1 - \rho_1 - \rho_2)} + E(B_3) = 3\frac{19}{30} = 3.63 \text{ hour} = 218 \text{ min}, \\ E(S) &= 1.24 \text{ hour} = 74.6 \text{ min.} \end{aligned}$$

(iii) In this situation we find

$$
\begin{aligned}
E(S_1) &= \frac{\rho_1 E(R_1) + \rho_2 E(R_2)}{1-\rho_1} + E(B_1) = \frac{49}{192} = 0.26 \text{ hour} = 15.3 \text{ min}, \\
E(S_2) &= \frac{\rho_1 E(R_1) + \rho_2 E(R_2)}{(1-\rho_1-\rho_2)(1-\rho_1)} + E(B_2) = \frac{37}{80} = 0.46 \text{ hour} = 27.8 \text{ min}, \\
E(S_3) &= \frac{\rho_1 E(R_1) + \rho_2 E(R_2) + \rho_3 E(R_3)}{(1-\rho_1-\rho_2-\rho_3)(1-\rho_1-\rho_2)} + \frac{E(B_3)}{1-\rho_1-\rho_2} = \frac{41}{10} = 4.1 \text{ hour} = 246 \text{ min}, \\
E(S) &= 1.27 \text{ hour} = 76.1 \text{ min}.
\end{aligned}
$$

Points

(i)	(ii)	(iii)
4	3	3