CS171 Introduction to Computer Science II

Recursion

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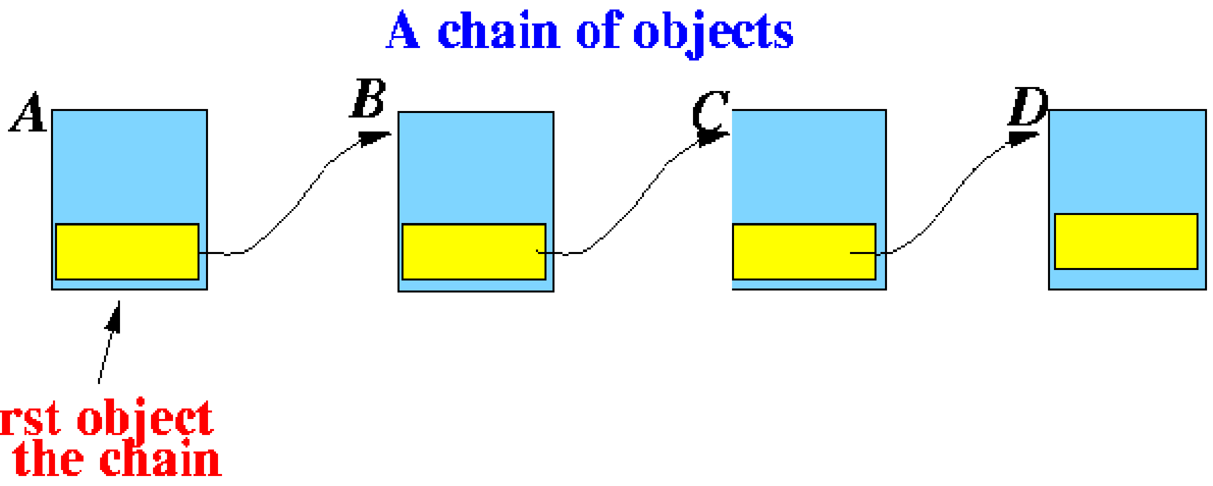
What we have learned so far

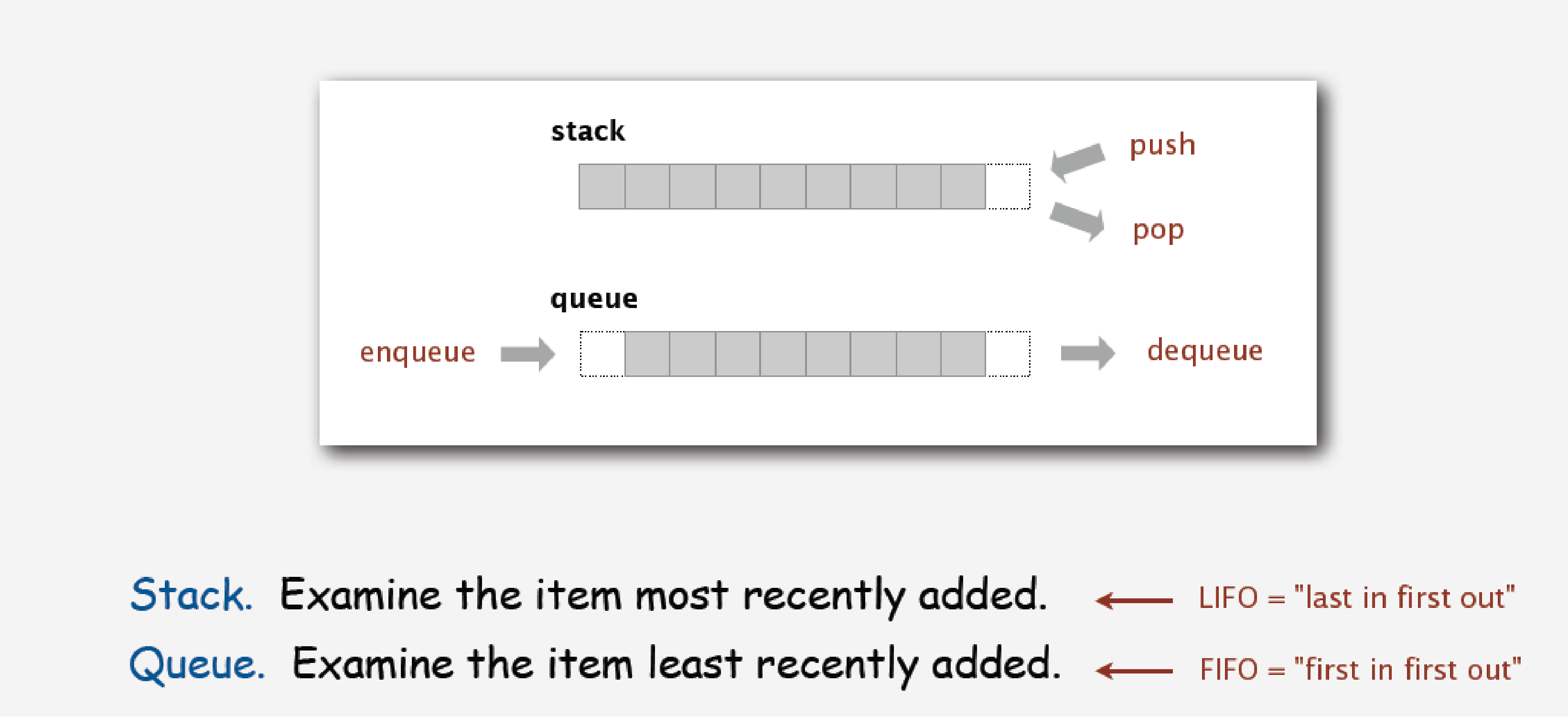
- Basic data structure
  - Arrays
  - Linked list
- Abstract data types
  - Stacks
  - Queues

Linked List

- A Linked List is a sequence of nodes chained together.
- Each **node**, element, or link contains a **data item**, and a **reference** to next node



Stacks and Queues



Stack. Examine the item most recently added. ← LIFO = "last in first out"

Queue. Examine the item least recently added. ← FIFO = "first in first out"

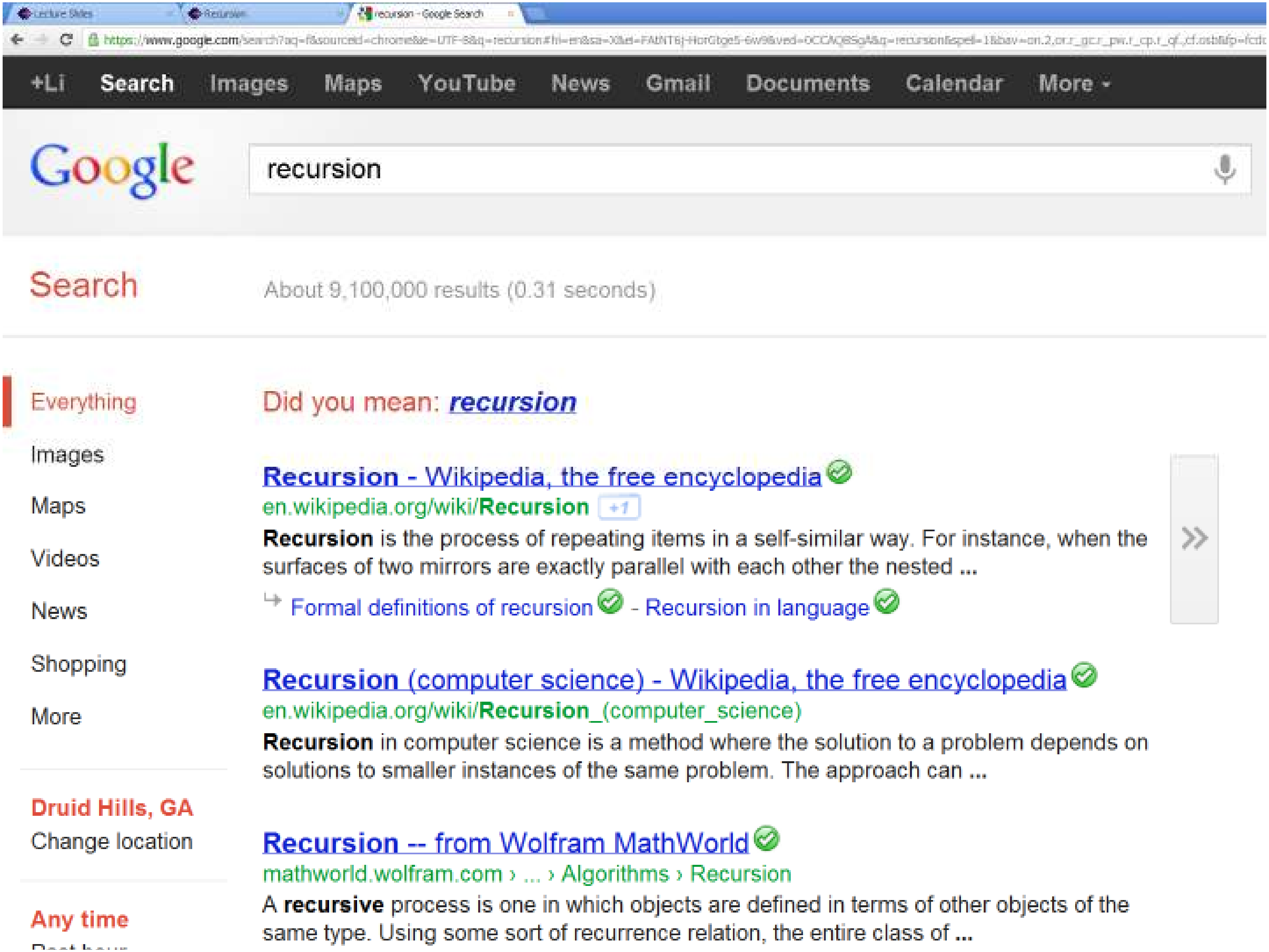
- Can be implemented by both (resizing) arrays and linked list

Today

- Quiz on stacks, queues, linked list
- Recursion

Recursion

- Recursion concept
- Examples
  - Factorial
  - Fibonacci
  - GCD
  - Recursive graph Htree
- Next lecture
  - Divide and conquer
  - Binary search
  - Tower of Hanoi
  - Cost analysis of recursive algorithms


Lecture Slides
Recursion
recursion - Google Search
https://www.google.com/search?aq=f&sourceid=chrome&ie=UTF-8&q=recursion#hl=en&sa=X&ei=FAtNT6j-HorGtge5-6w9&ved=0CCAQBSgA&q=recursion&spell=1&bav=on.2,or.r_gc.r_pw.r_cp.r_qf.,cf.osb&fp=fcdc
+Li
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Images
Maps
YouTube
News
Gmail
Documents
Calendar
More
Google
Search
Everything
Images
Maps
Videos
News
Shopping
More
Druid Hills, GA
Change location
Any time
Past hour
+1

recursion

About 9,100,000 results (0.31 seconds)

Did you mean: *recursion*

Recursion - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/**Recursion**
Recursion is the process of repeating items in a self-similar way. For instance, when the surfaces of two mirrors are exactly parallel with each other the nested ...
Formal definitions of recursion - Recursion in language

Recursion (computer science) - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/**Recursion**_(computer_science)
Recursion in computer science is a method where the solution to a problem depends on solutions to smaller instances of the same problem. The approach can ...

Recursion -- from Wolfram MathWorld
mathworld.wolfram.com › ... › Algorithms › Recursion
A **recursive** process is one in which objects are defined in terms of other objects of the same type. Using some sort of recurrence relation, the entire class of ...

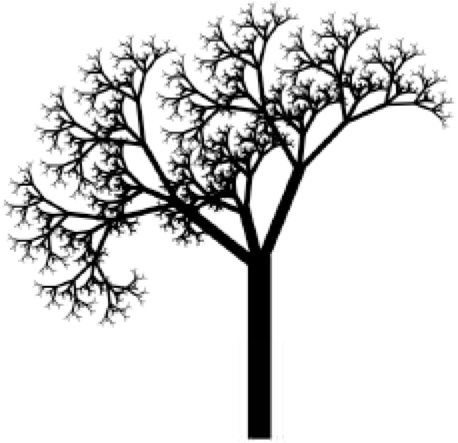
What is recursion?

Overview

What is recursion? When one function calls itself directly or indirectly.

Why learn recursion?

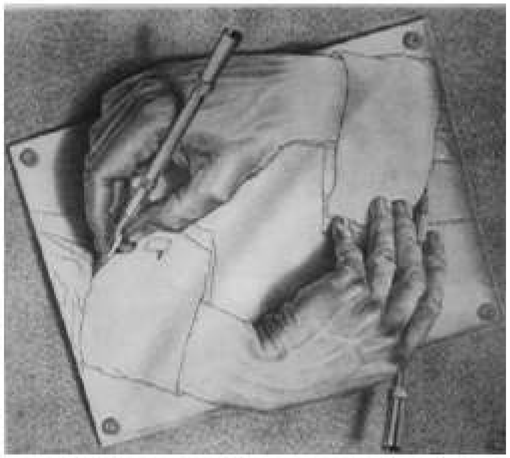
- New mode of thinking.
- Powerful programming paradigm.



Many computations are naturally self-referential.

- Mergesort, FFT, gcd, depth-first search.
- Linked data structures.
- A folder contains files and other folders.

Closely related to mathematical induction.



Reproductive Parts

Factorial

N! = N*(N-1)*(N-2)*…..* 2 * 1

```
int fact (int N)
{
   if (N==0)
        return 1;
   else
        return (N * fact (N-1));
}
```

Recursive Method

- A method that calls itself (direct recursion)

```
void  recursiveMethod()    {
   …    …
   recursiveMethod();
   …  …
}
```

Recursive Method

- A method that calls itself (direct recursion)
- Every recursive method must have a base case that is not recursive

```
void recursiveMethod() {
     …
     if (base case) {
            … …
     }
     else {
            … …
            recursiveMethod();
            … …
     }
 }
```

Better version of recursion definition

Recursion

n. If you still don't get it, see Recursion.

Recursion

- A method calls itself
  - Calls a “clone” of itself to solve a **smaller** problem
  - Buck Passing
- Must have a base case
  - The buck stops here! (does **not** call the method)

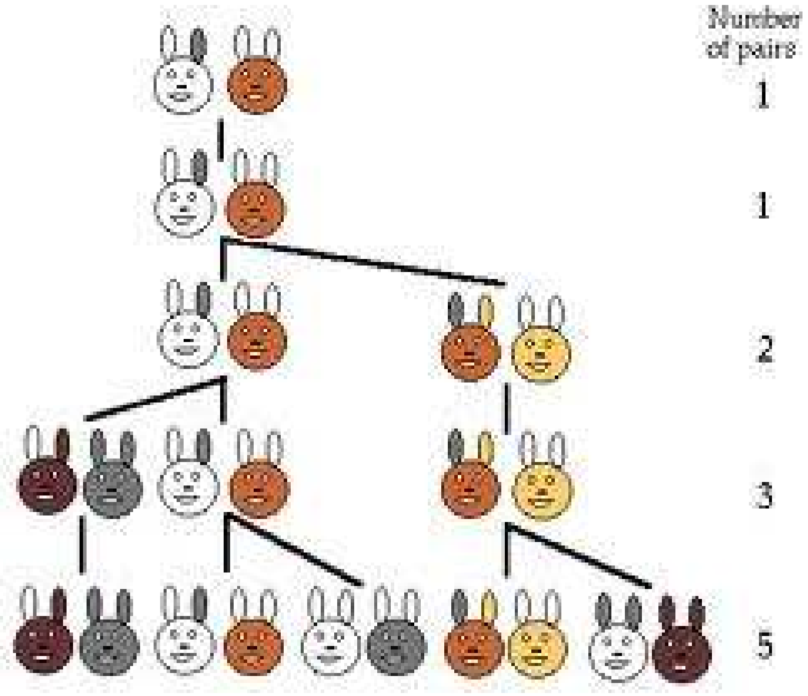
Example: Fibonacci Numbers

- Recursive formula:

$$F(n) = F(n-1) + F(n-2)$$

$$F(0) = 0, \;\; F(1) = 1$$

- 0, 1, 1, 2, 3, 5, 8, 13, …..



Fibonacci Numbers: Java Code

```
int F(int n)
{
   if (n==0)
        return 0;
   else if (n==1)
        return 1;
   else
        return F(n-1)+F(n-2);
}
```

Greatest Common Divisor

Gcd. Find largest integer that evenly divides into p and q.

Ex. gcd(4032, 1272) = 24.

$$\begin{aligned} 4032 &= 2^6 \times 3^2 \times 7^1 \\ 1272 &= 2^3 \times 3^1 \times 53^1 \\ \text{gcd} &= 2^3 \times 3^1 = 24 \end{aligned}$$

Applications.

- Simplify fractions: 1272/4032 = 53/168.
- RSA cryptosystem.

Greatest Common Divisor

Gcd. Find largest integer d that evenly divides into p and q.

Euclid's algorithm. [Euclid 300 BCE]

$$\gcd(p, q) = \begin{cases} p & \text{if } \; q = 0 & \quad \leftarrow \text{ base case} \\ \gcd(q, \; p \;\%\; q) & \text{otherwise} & \quad \leftarrow \text{ reduction step, converges to base case} \end{cases}$$

```
gcd(4032, 1272)   = gcd(1272, 216)
                  = gcd(216, 192)
                  = gcd(192, 24)
                  = gcd(24, 0)
                  = 24.
```

4032 = 3 x 1272 + 216

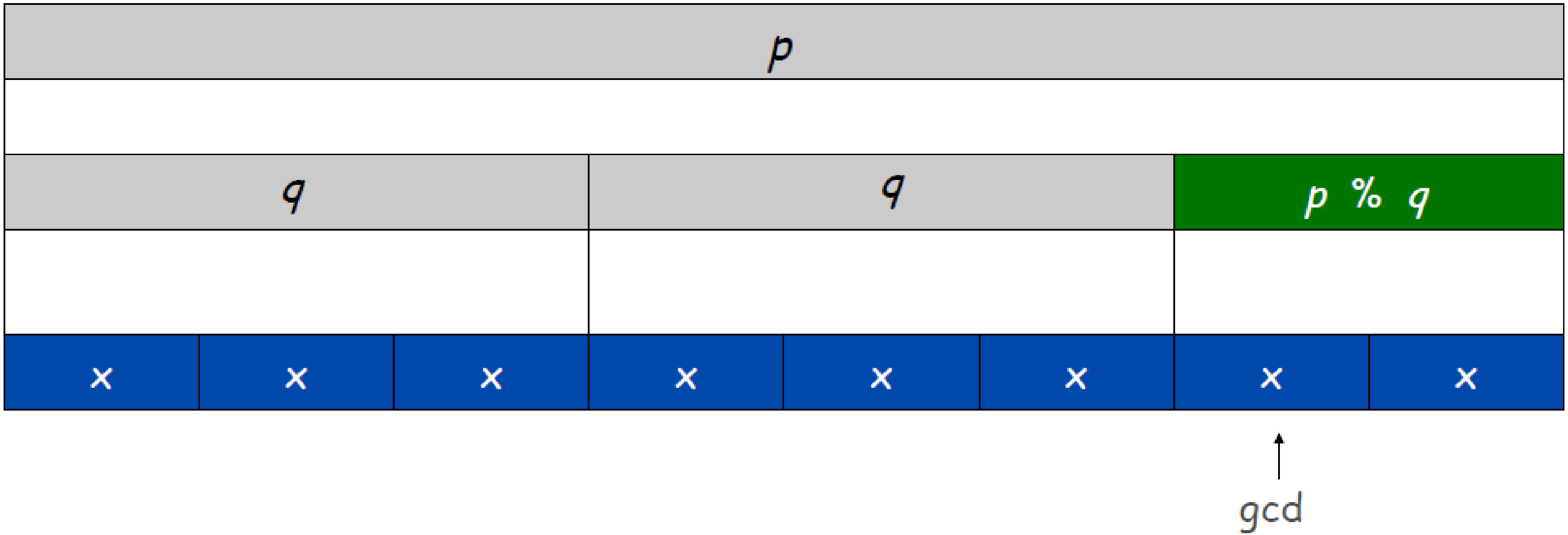
Greatest Common Divisor

Gcd. Find largest integer d that evenly divides into p and q.

$$\gcd(p, q) = \begin{cases} p & \text{if } q = 0 \\ \gcd(q,\ p \ \%\ q) & \text{otherwise} \end{cases}$$

← base case

← reduction step, converges to base case



p = 8x
q = 3x
gcd(p, q) = x

Greatest Common Divisor

Gcd. Find largest integer d that evenly divides into p and q.

$$\gcd(p, q) = \begin{cases} p & \text{if } q = 0 \\ \gcd(q,\ p \ \% \ q) & \text{otherwise} \end{cases}$$

← base case

← reduction step,
converges to base case

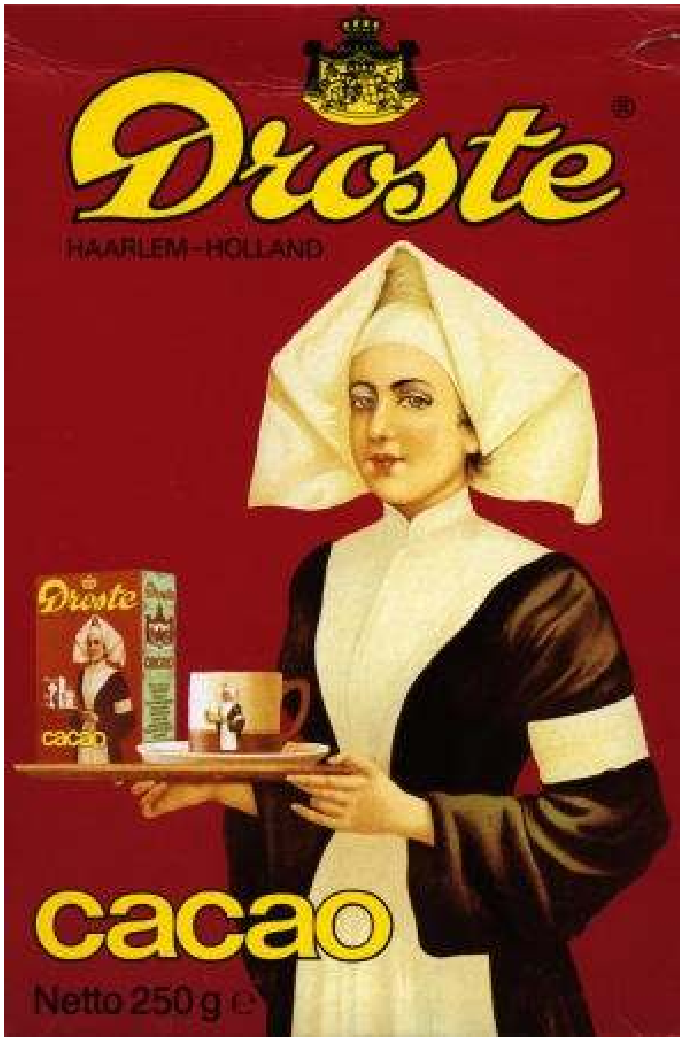
Java implementation.

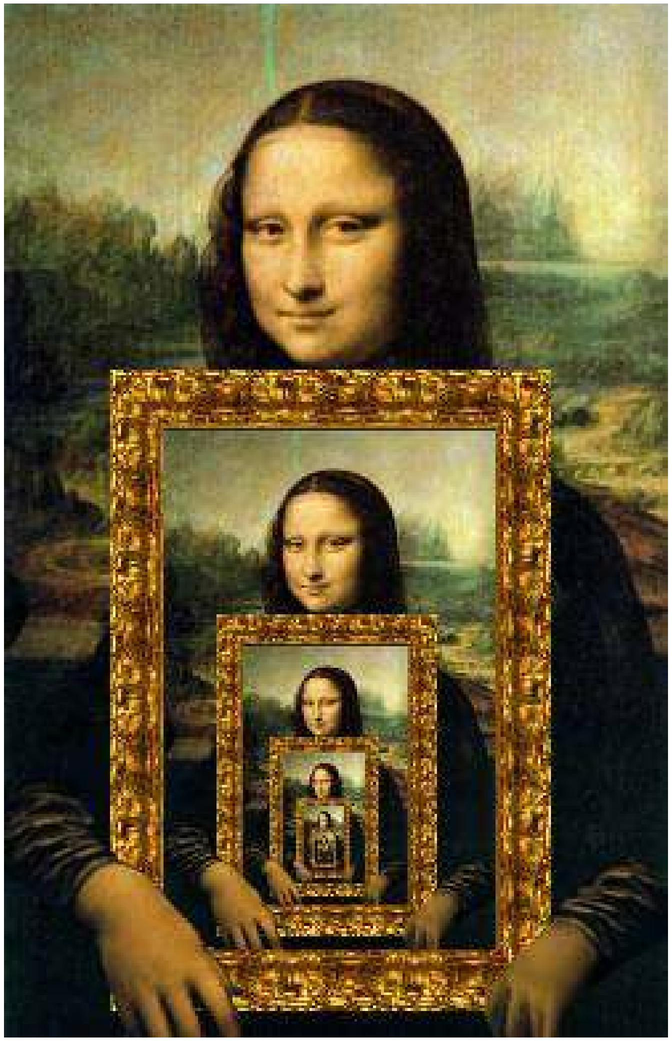
```java
public static int gcd(int p, int q) {
   if (q == 0) return p;
   else return gcd(q, p % q);
}
```

← base case

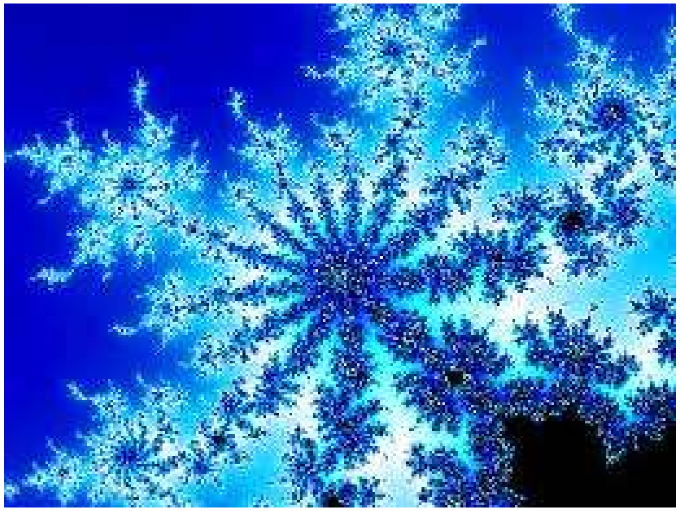
← reduction step

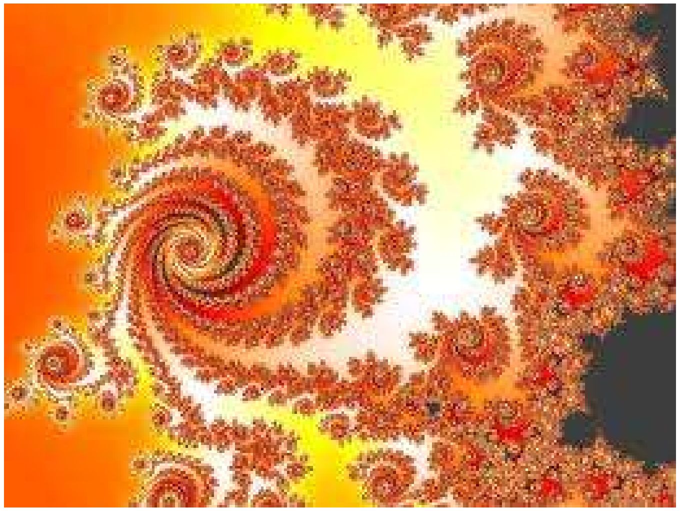
Visual Recursion

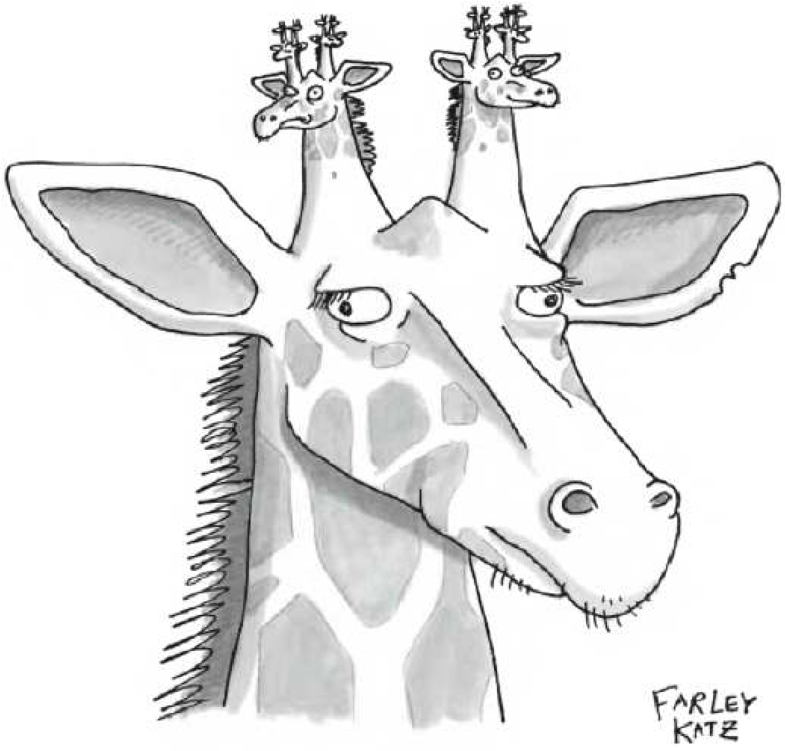


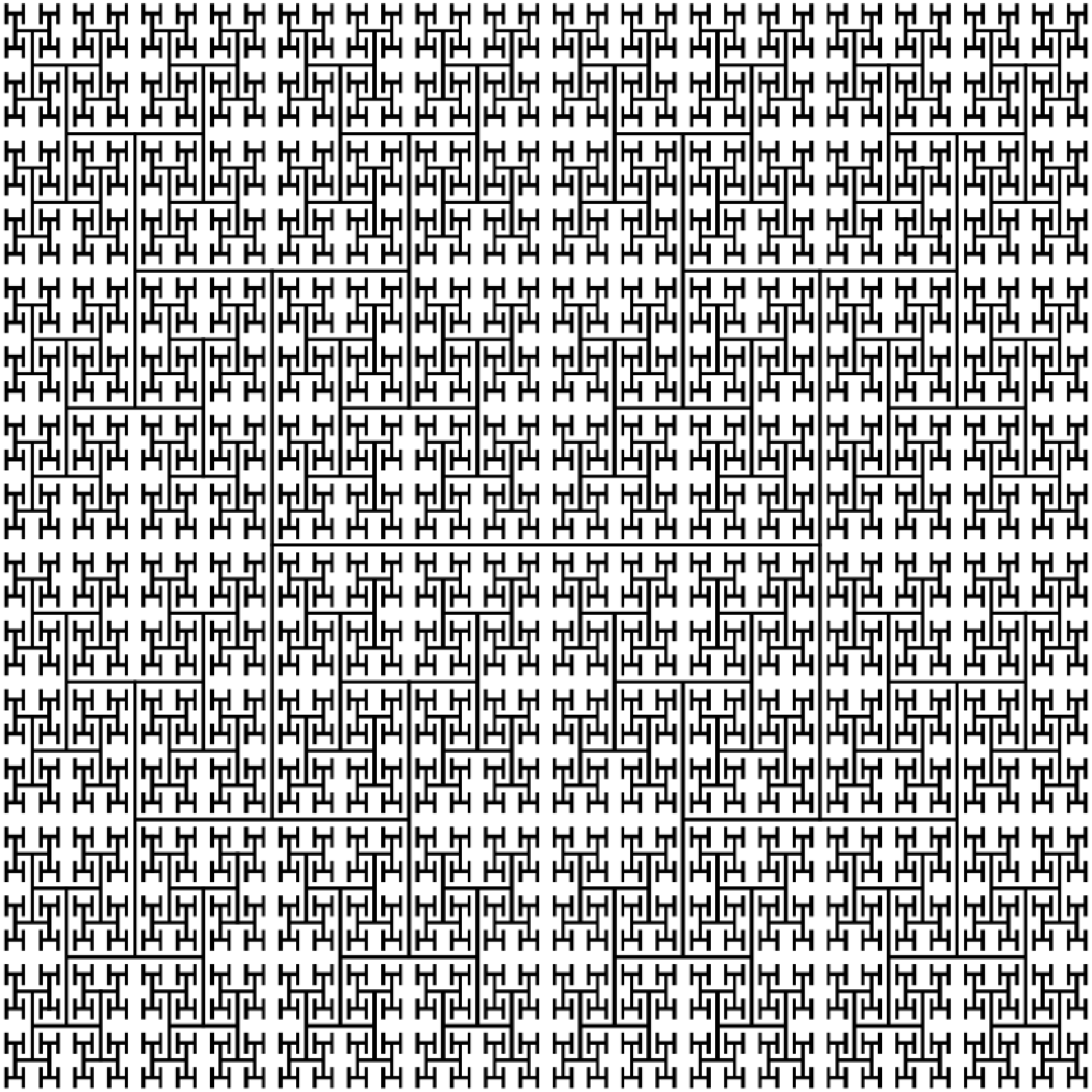


Fractals







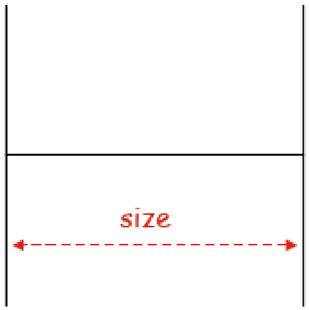


Htree

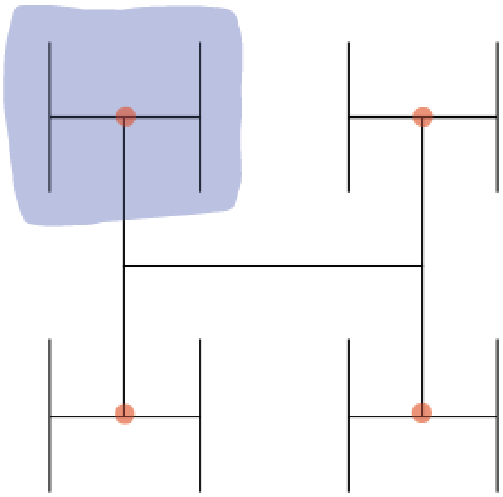
H-tree of order n.

- Draw an H.
- Recursively draw 4 H-trees of order n-1, one connected to each tip.

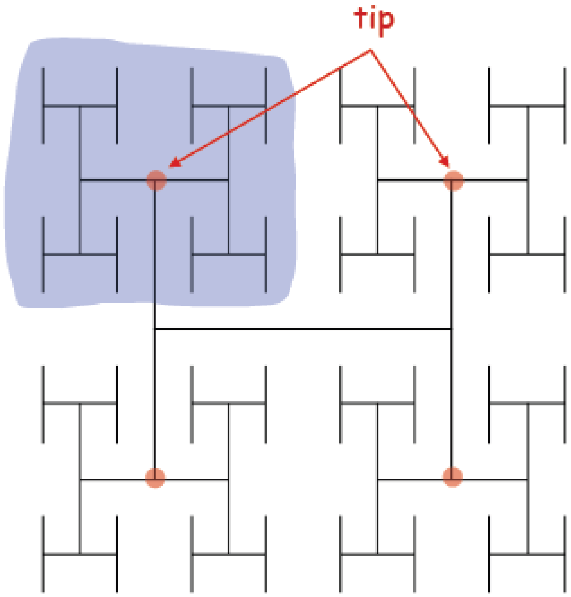
and half the size



order 1



order 2



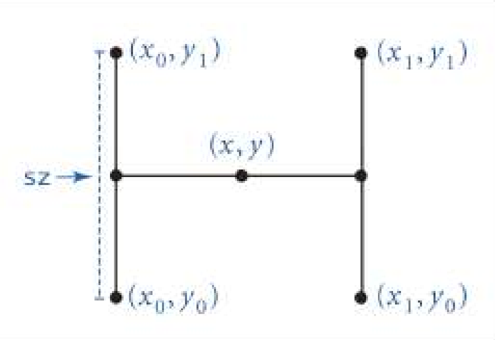
order 3

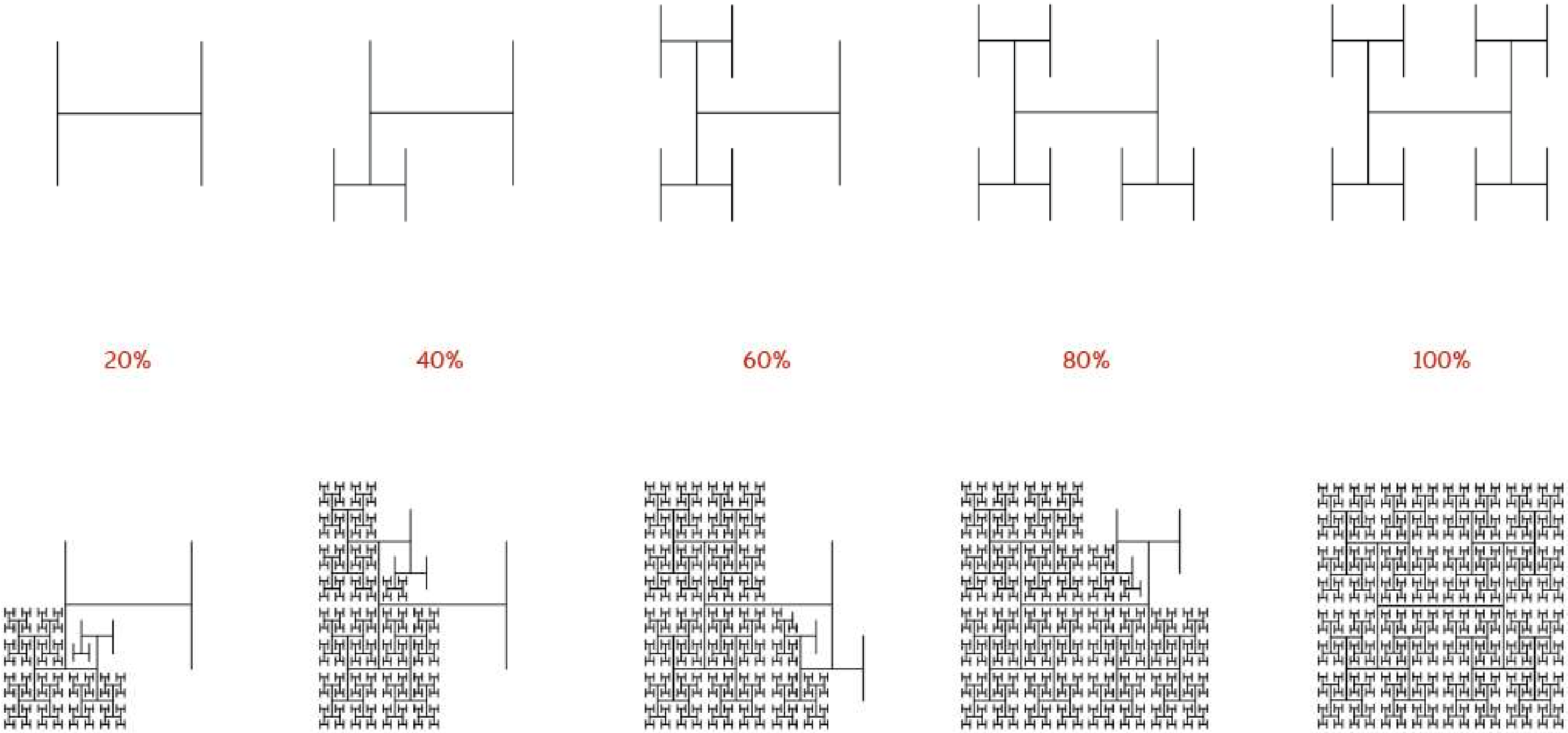
Htree in Java

```java
public class Htree {
   public static void draw(int n, double sz, double x, double y) {
      if (n == 0) return;
      double x0 = x - sz/2, x1 = x + sz/2;
      double y0 = y - sz/2, y1 = y + sz/2;

      StdDraw.line(x0,  y, x1,  y);
      StdDraw.line(x0, y0, x0, y1);
      StdDraw.line(x1, y0, x1, y1);

      draw(n-1, sz/2, x0, y0);
      draw(n-1, sz/2, x0, y1);
      draw(n-1, sz/2, x1, y0);
      draw(n-1, sz/2, x1, y1);
   }

   public static void main(String[] args) {
      int n = Integer.parseInt(args[0]);
      draw(n, .5, .5, .5);
   }
}
```

← draw the H, centered on (x, y)

← recursively draw 4 half-size Hs

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Recursion

- Recursive method
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  - Factorial
  - Fibonacci
  - GCD
  - Recursive graph Htree
- Next lecture
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  - Binary search
  - Tower of Hanoi
  - Cost analysis of recursive algorithms