CS 171: Introduction to Computer Science II

Binary Search Trees (cont.)

Binary Search Trees

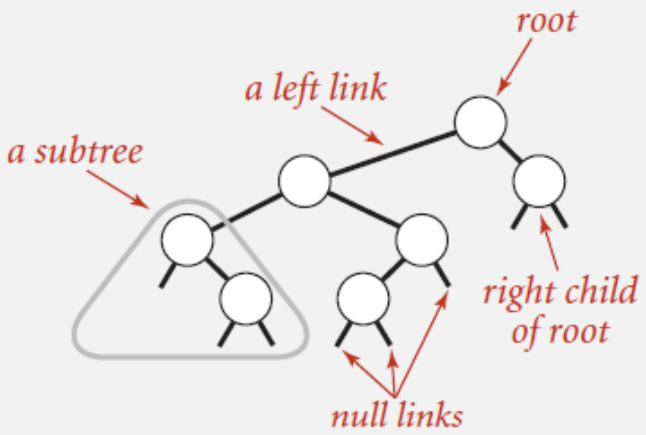
- Definitions and terminologies
- Search and insert
- Traversal
- Ordered operations
- Delete

Binary search trees

Definition. A BST is a binary tree in symmetric order.

A binary tree is either:

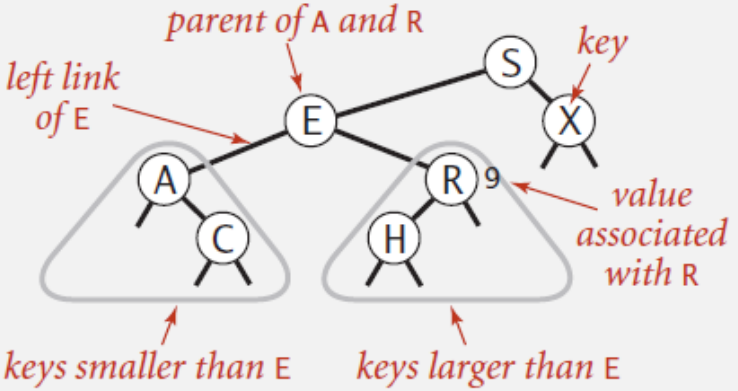
- Empty.
- Two disjoint binary trees (left and right).



Anatomy of a binary tree

Symmetric order. Each node has a key, and every node's key is:

- Larger than all keys in its left subtree.
- Smaller than all keys in its right subtree.



Anatomy of a binary search tree

BST representation in Java

Java definition. A BST is a reference to a root `Node`.

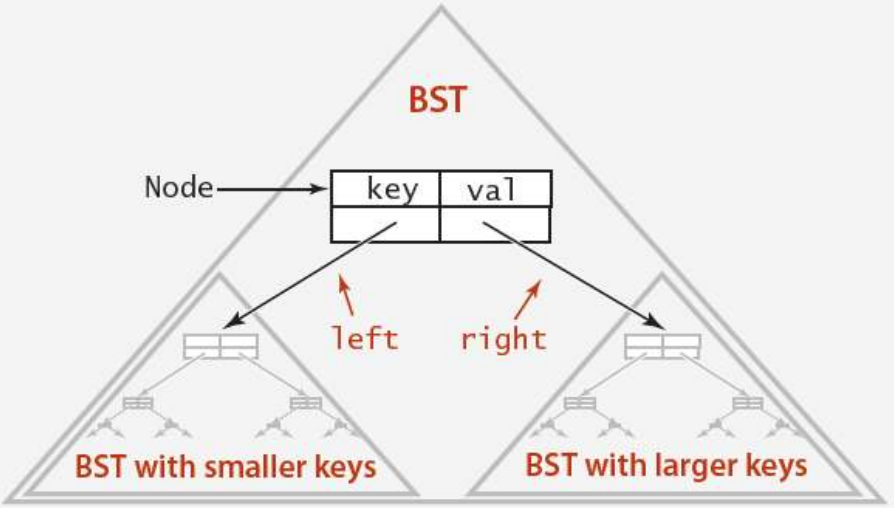
A `Node` is comprised of four fields:

- A `Key` and a `Value`.
- A reference to the left and right subtree.

smaller keys larger keys

```
private class Node
{
   private Key key;
   private Value val;
   private Node left, right;
   public Node(Key key, Value val)
   {
      this.key = key;
      this.val = val;
   }
}
```

`Key` and `Value` are generic types; `Key` is `Comparable`



Binary search tree

ST implementations: summary

implementation	guarantee		average case		ordered ops?	operations on keys
	search	insert	search hit	insert		
sequential search (unordered list)	N	N	N/2	N	no	`equals()`
binary search (ordered array)	lg N	N	lg N	N/2	yes	`compareTo()`
BST	N	N	1.39 lg N	1.39 lg N	?	`compareTo()`

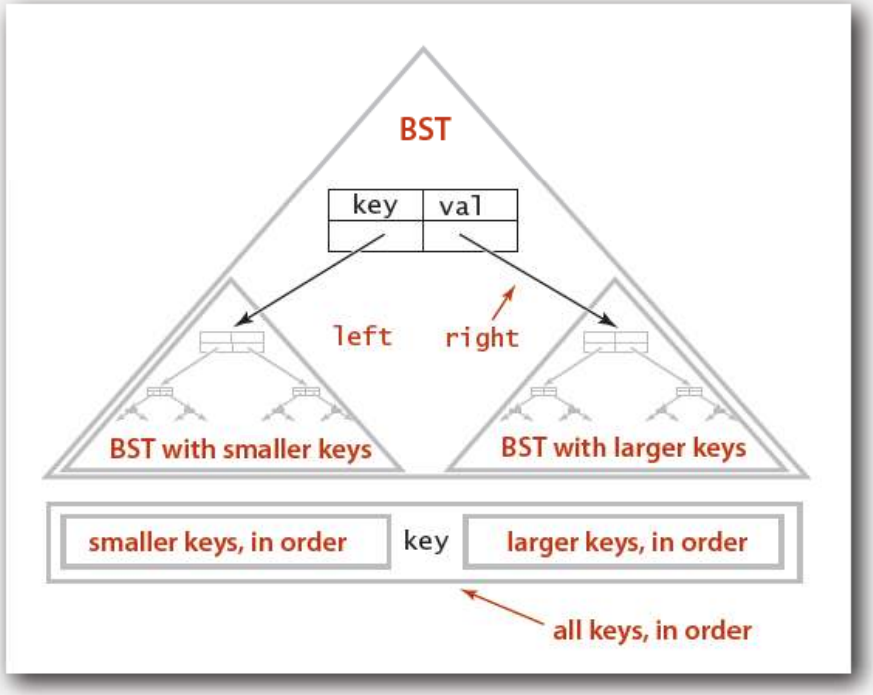
Traversal

- In-order
  - Left subtree, current node, right subtree
- Pre-order
  - Current node, left subtree, right subtree
- Post-order
  - Left subtree, right subtree, current node

Inorder traversal

- Traverse left subtree.
- Enqueue key.
- Traverse right subtree.

```
public Iterable<Key> keys()
{
    Queue<Key> q = new Queue<Key>();
    inorder(root, q);
    return q;
}

private void inorder(Node x, Queue<Key> q)
{
   if (x == null) return;
   inorder(x.left, q);
   q.enqueue(x.key);
   inorder(x.right, q);
}
```

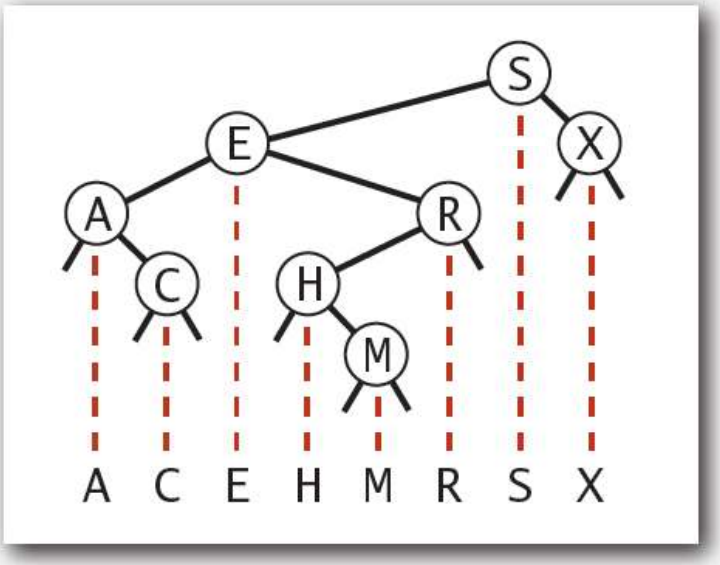


Property. Inorder traversal of a BST yields keys in ascending order.

Inorder traversal

- Traverse left subtree.
- Enqueue key.
- Traverse right subtree.

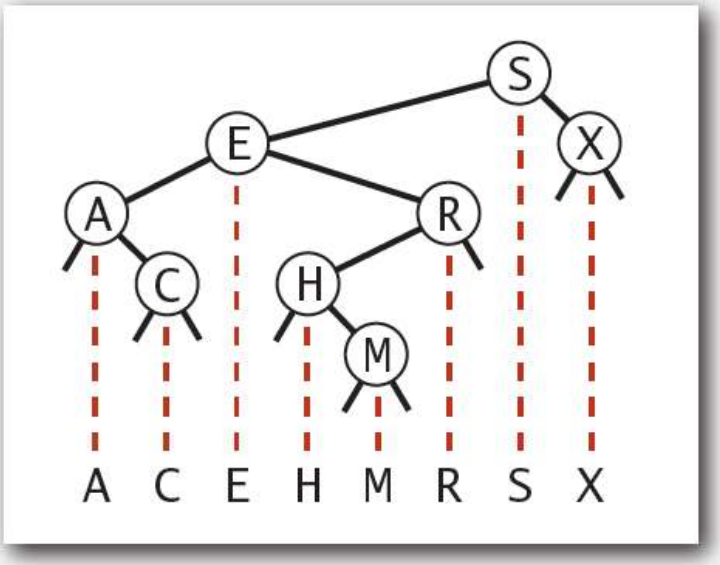
recursive calls	queue	function call stack
`inorder(S)`		S
`  inorder(E)`		S E
`    inorder(A)`		S E A
`      enqueue A`	A	
`      inorder(C)`		S E A C
`        enqueue C`	C	
`    enqueue E`	E	
`    inorder(R)`		S E R
`      inorder(H)`		S E R H
`        enqueue H`	H	
`        inorder(M)`		S E R H M
`          enqueue M`	M	
`      enqueue R`	R	
`  enqueue S`	S	
`  inorder(X)`		S X
`    enqueue X`	X	



Traversal

- In-order

  A C E H M R S X
- Pre-order?
- Post-order?



Traversal

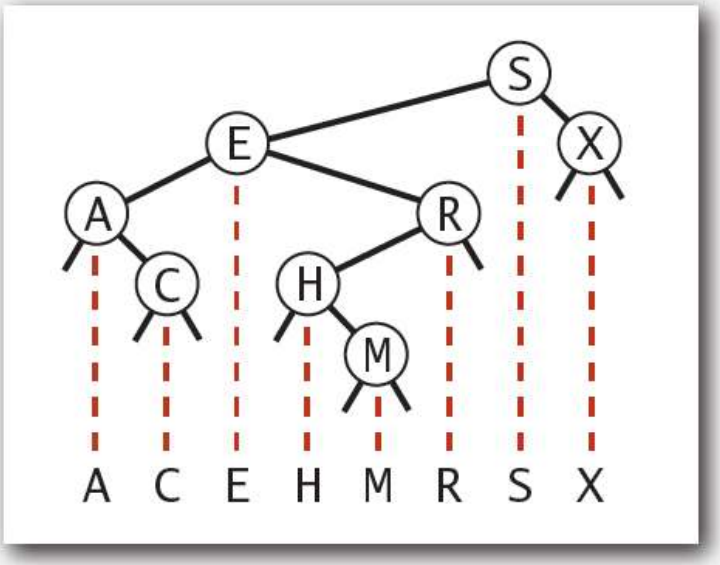
- In-order

  A C E H M R S X
- Pre-order

  S E A C R H M X
- Post-order

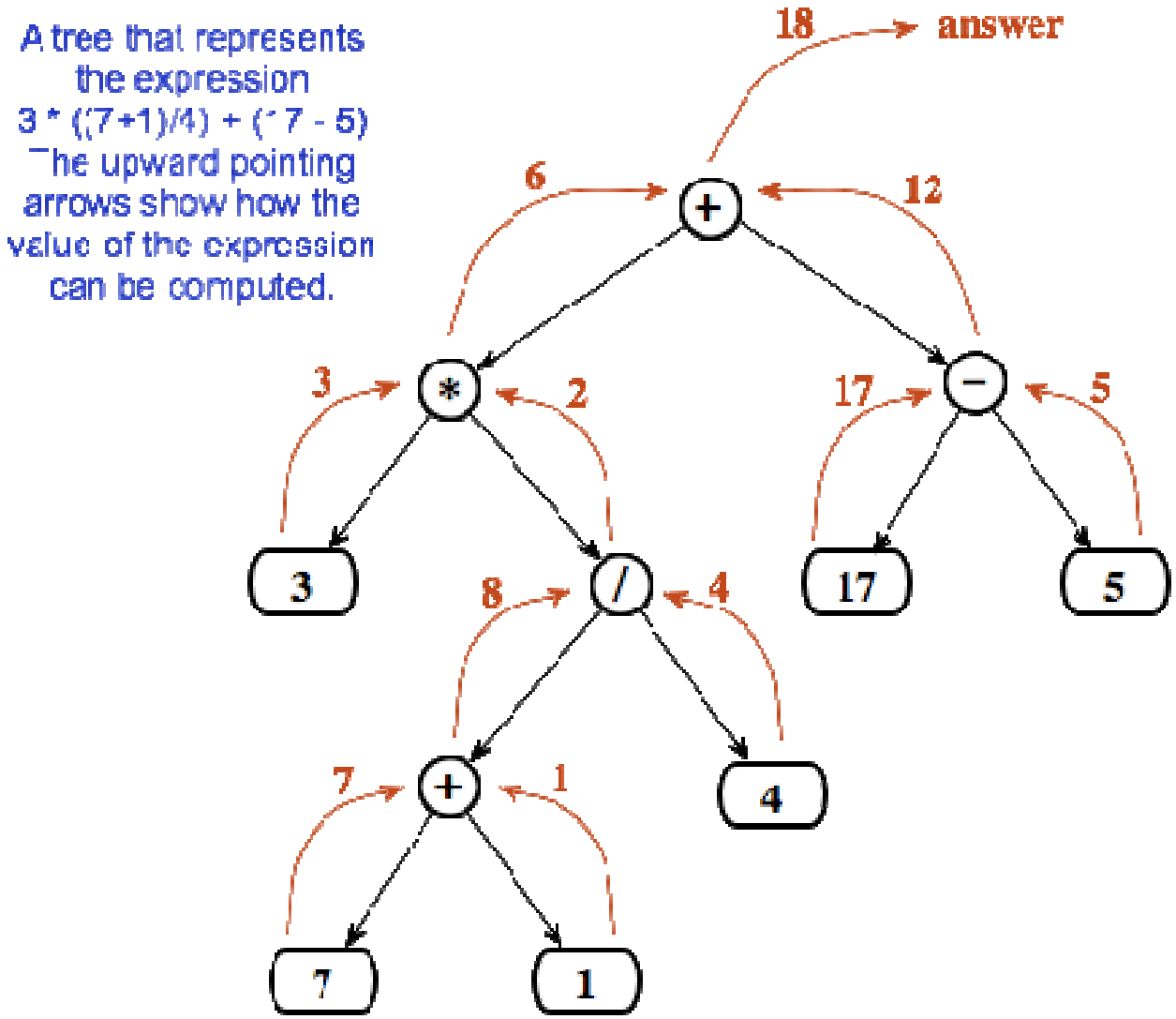
  C A M H R E X S

- How to visit the nodes in descending order?
- What's the use of pre-order and post-order traversal?



Expression Tree

- Post-order traversal results in postfix notation
- Pre-order traversal results in prefix notation



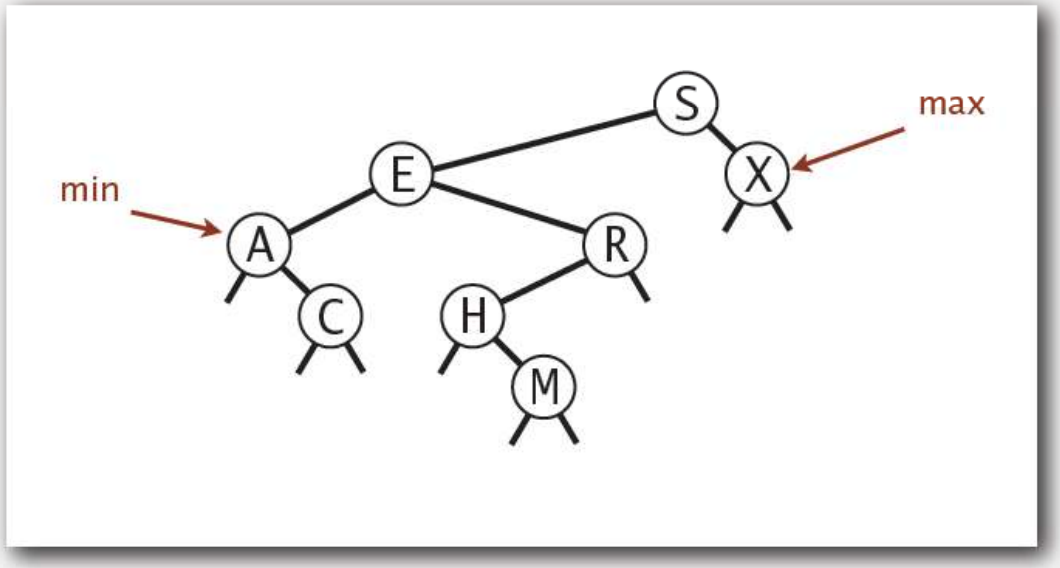
Binary Search Trees

- Definitions and terminologies
- Search and insert
- Traversal
- Ordered operations
  - Minimum and maximum
  - Rank: how many keys < k?
  - Select: key of given rank
- Delete

Minimum and maximum

Minimum. Smallest key in table.

Maximum. Largest key in table.

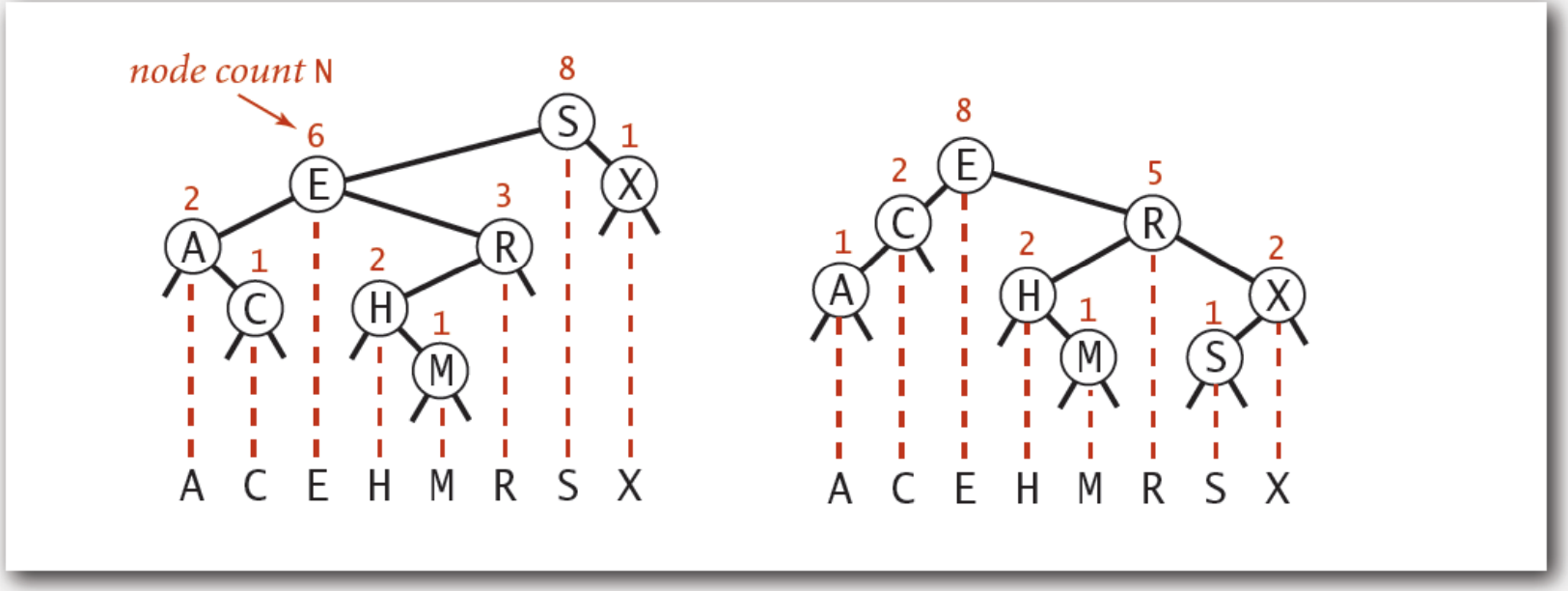


Q. How to find the min / max?

Subtree counts

In each node, we store the number of nodes in the subtree rooted at that node.

To implement `size()`, return the count at the root.



Remark. This facilitates efficient implementation of `rank()` and `select()`.

BST implementation: subtree counts

```
private class Node
{
   private Key key;
   private Value val;
   private Node left;
   private Node right;
   private int N;
}
```

number of nodes in subtree

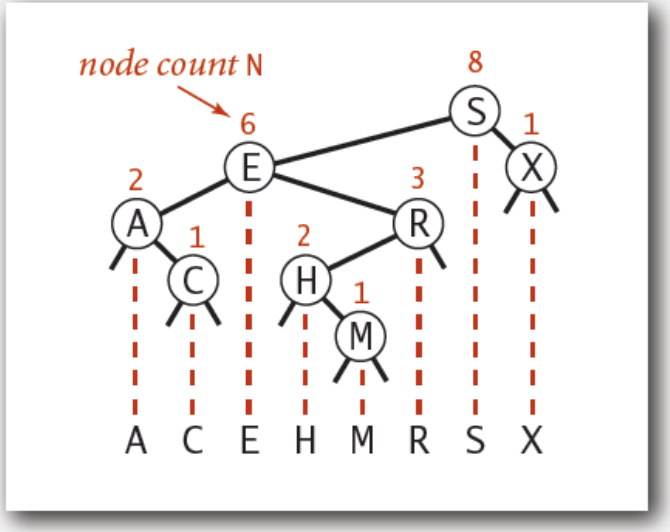
```
public int size()
{  return size(root);  }

private int size(Node x)
{
   if (x == null) return 0;
   return x.N;
}
```

```
private Node put(Node x, Key key, Value val)
{
   if (x == null) return new Node(key, val);
   int cmp = key.compareTo(x.key);
   if      (cmp  < 0) x.left  = put(x.left,  key, val);
   else if (cmp  > 0) x.right = put(x.right, key, val);
   else if (cmp == 0) x.val = val;
   x.N = 1 + size(x.left) + size(x.right);
   return x;
}
```

Rank

Rank. How many keys < k ?

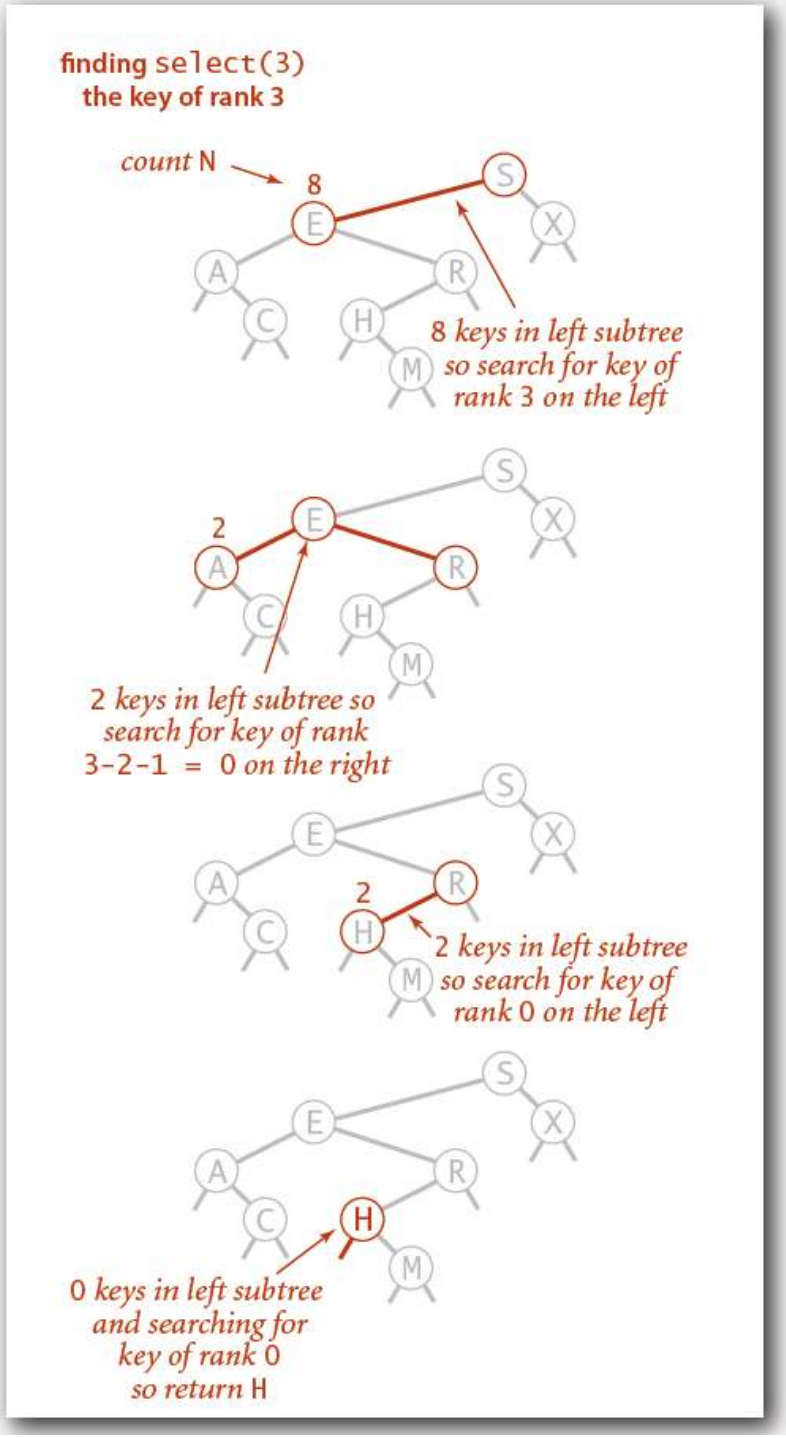
Easy recursive algorithm (4 cases!)




```
public int rank(Key key)
{  return rank(key, root);  }

private int rank(Key key, Node x)
{
   if (x == null) return 0;
   int cmp = key.compareTo(x.key);
   if      (cmp  < 0) return rank(key, x.left);
   else if (cmp  > 0) return 1 + size(x.left) + rank(key, x.right);
   else if (cmp == 0) return size(x.left);
}
```

Select. Key of given rank.

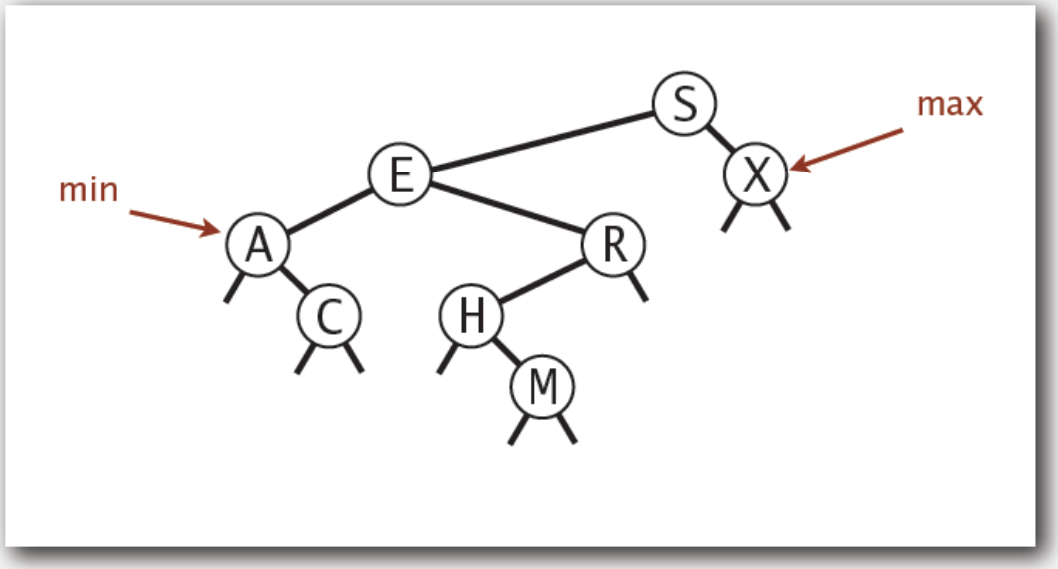
```
public Key select(int k)
{
    if (k < 0) return null;
    if (k >= size()) return null;
    Node x = select(root, k);
    return x.key;
}

private Node select(Node x, int k)
{
   if (x == null) return null;
   int t = size(x.left);
   if      (t  > k)
      return select(x.left,  k);
   else if (t  < k)
      return select(x.right, k-t-1);
   else if (t == k)
      return x;
}
```



Binary Search Trees

- Definitions and terminologies
- Search and insert
- Traversal
- Ordered operations
- Delete
  - Delete minimum and maximum
  - Delete a given key

Delete minimum

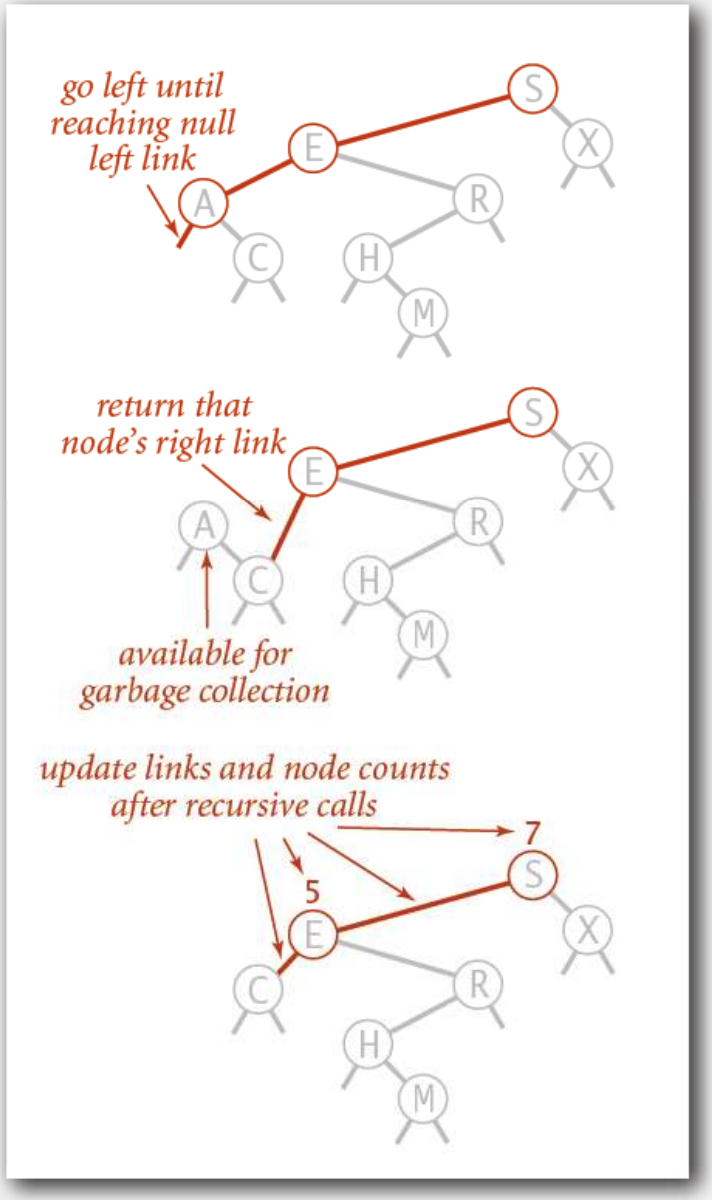


Deleting the minimum

To delete the minimum key:

- Go left until finding a node with a null left link.
- Replace that node by its right link.
- Update subtree counts.

```
public void deleteMin()
{  root = deleteMin(root);  }

private Node deleteMin(Node x)
{
   if (x.left == null) return x.right;
   x.left = deleteMin(x.left);
   x.N = 1 + size(x.left) + size(x.right);
   return x;
}
```

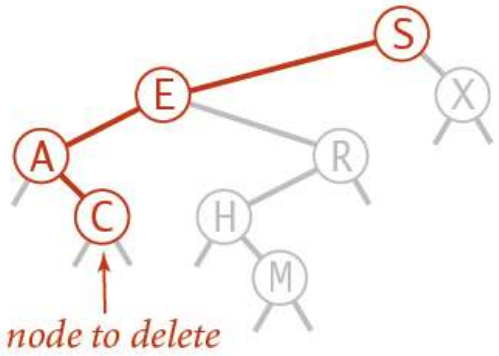


Hibbard deletion

To delete a node with key k: search for node t containing key k.

Case 0. [0 children]

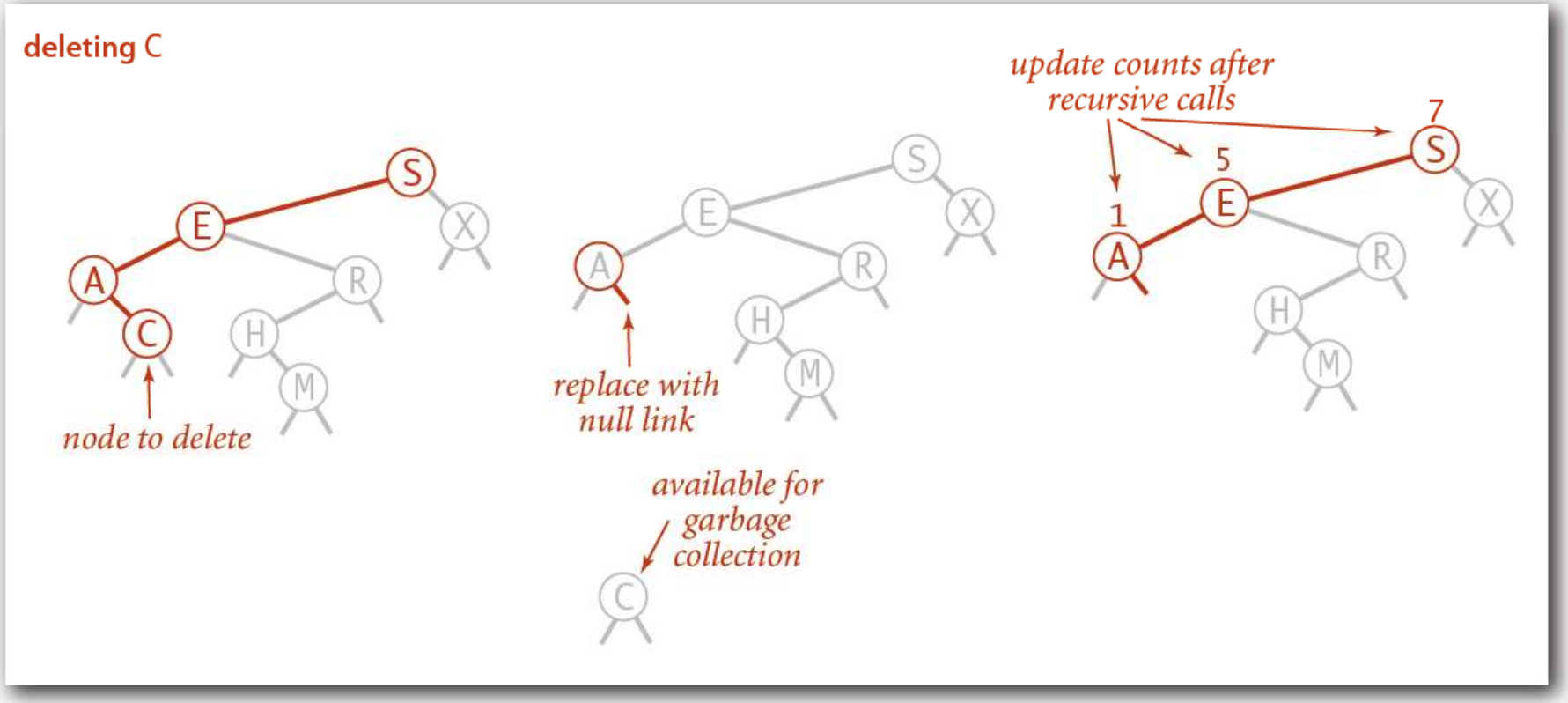
deleting C



Hibbard deletion

To delete a node with key k: search for node t containing key k.

Case 0. [0 children] Delete t by setting parent link to null.

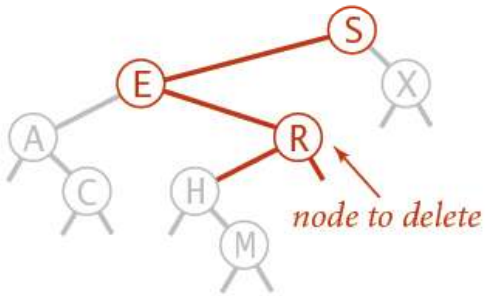


Hibbard deletion

To delete a node with key k: search for node t containing key k.

Case 1. [1 child]

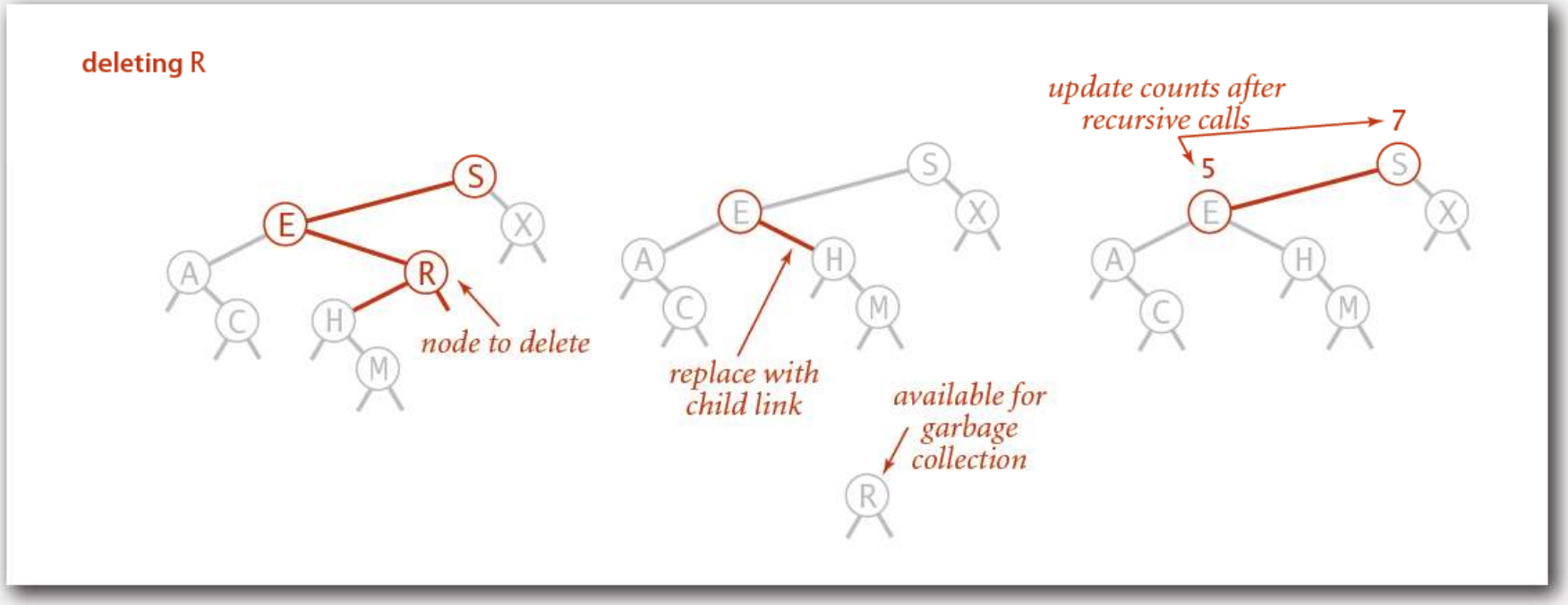
deleting R



Hibbard deletion

To delete a node with key k: search for node t containing key k.

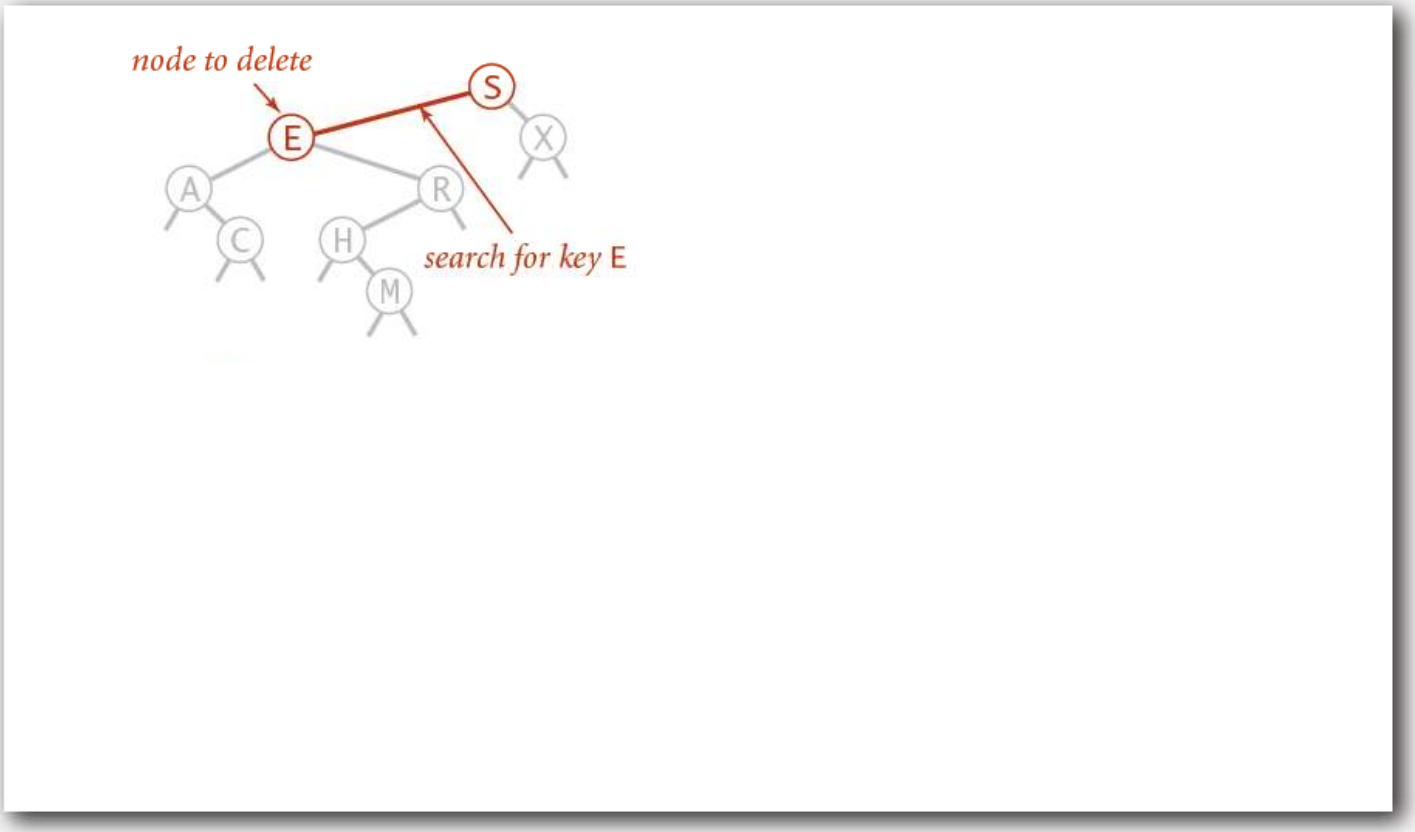
Case 1. [1 child] Delete t by replacing parent link.



Hibbard deletion

To delete a node with key k: search for node t containing key k.

Case 2. [2 children]

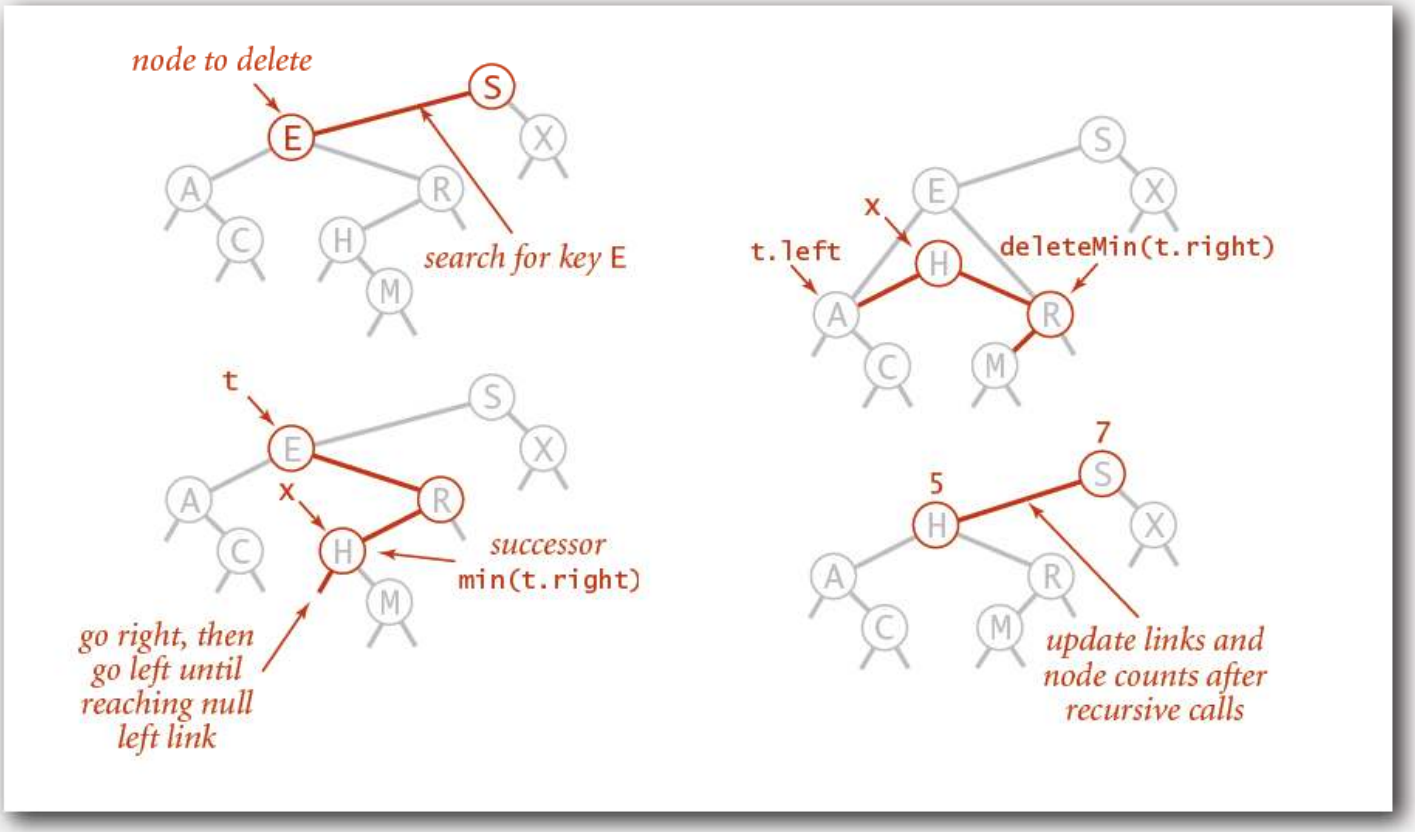


Hibbard deletion

To delete a node with key k: search for node t containing key k.

Case 2. [2 children]

- Find successor x of t. ⟵ x has no left child
- Delete the minimum in t's right subtree. ⟵ but don't garbage collect x
- Put x in t's spot. ⟵ still a BST



Hibbard deletion: Java implementation

```
public void delete(Key key)
{  root = delete(root, key);  }

private Node delete(Node x, Key key) {
   if (x == null) return null;
   int cmp = key.compareTo(x.key);
   if      (cmp < 0) x.left  = delete(x.left,  key);
   else if (cmp > 0) x.right = delete(x.right, key);
   else {
      if (x.right == null) return x.left;

      Node t = x;
      x = min(t.right);
      x.right = deleteMin(t.right);
      x.left = t.left;
   }
   x.N = size(x.left) + size(x.right) + 1;
   return x;
}
```

search for key

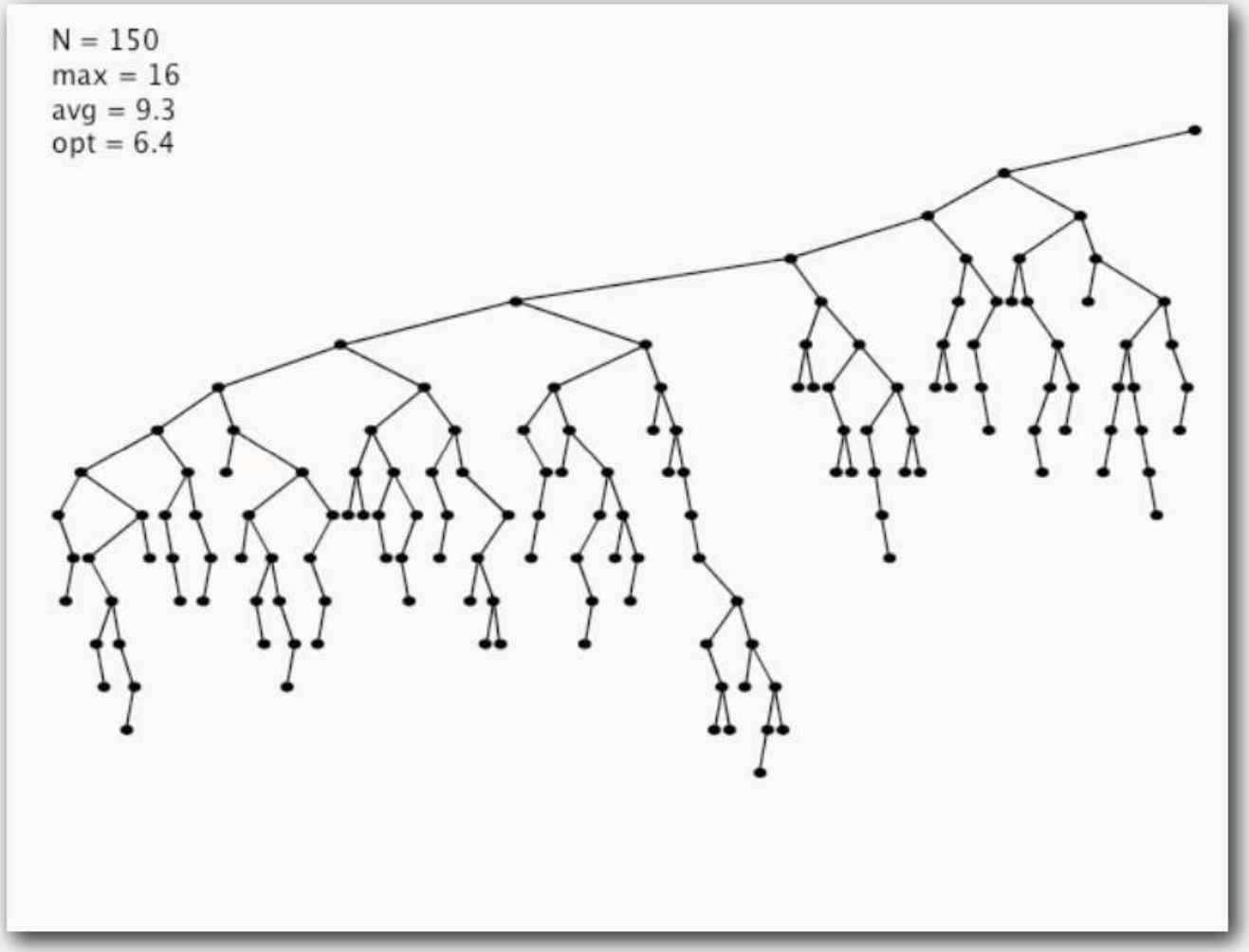
no right child

replace with successor

update subtree counts

Hibbard deletion: analysis

Unsatisfactory solution. Not symmetric.



Surprising consequence. Trees not random (!) $\Rightarrow$ sqrt (N) per op.

Longstanding open problem. Simple and efficient delete for BSTs.

ST implementations: summary

implementation	guarantee			average case			ordered iteration?	operations on keys
	search	insert	delete	search hit	insert	delete		
sequential search (linked list)	N	N	N	N/2	N	N/2	no	`equals()`
binary search (ordered array)	lg N	N	N	lg N	N/2	N/2	yes	`compareTo()`
BST	N	N	N	1.39 lg N	1.39 lg N	$\sqrt{N}$	yes	`compareTo()`

other operations also become $\sqrt{N}$ if deletions allowed

Binary Search Trees

- Definitions and terminologies
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- Traversal
- Ordered operations
- Delete

- Balanced search trees (Amy Shannon)