5.2 Tries

- R-way tries
- ternary search tries
- character-based operations

Review: summary of the performance of symbol-table implementations

Frequency of operations.

implementation	typical case			ordered operations	operations on keys
	search	insert	delete		
red-black BST	1.00 lg N	1.00 lg N	1.00 lg N	yes	`compareTo()`
hash table	1 †	1 †	1 †	no	`equals()` `hashcode()`

† under uniform hashing assumption

Q. Can we do better?

A. Yes, if we can avoid examining the entire key, as with string sorting.

String symbol table basic API

String symbol table. Symbol table specialized to string keys.

```
public class StringST<Value>

               StringST()                          create an empty symbol table
          void put(String key, Value val)          put key-value pair into the symbol table
         Value get(String key)                     return value paired with given key
          void delete(String key)                  delete key and corresponding value
               ⋮
```

Goal. Faster than hashing, more flexible than BSTs.

String symbol table implementations cost summary

	character accesses (typical case)				dedup	
implementation	search hit	search miss	insert	space (references)	moby.txt	actors.txt
red-black BST	$L + c \lg^2 N$	$c \lg^2 N$	$c \lg^2 N$	$4N$	1.40	97.4
hashing	L	L	L	$4N$ to $16N$	0.76	40.6

Parameters
- N = number of strings
- L = length of string
- R = radix

file	size	words	distinct
moby.txt	1.2 MB	210 K	32 K
actors.txt	82 MB	11.4 M	900 K

Challenge. Efficient performance for string keys.

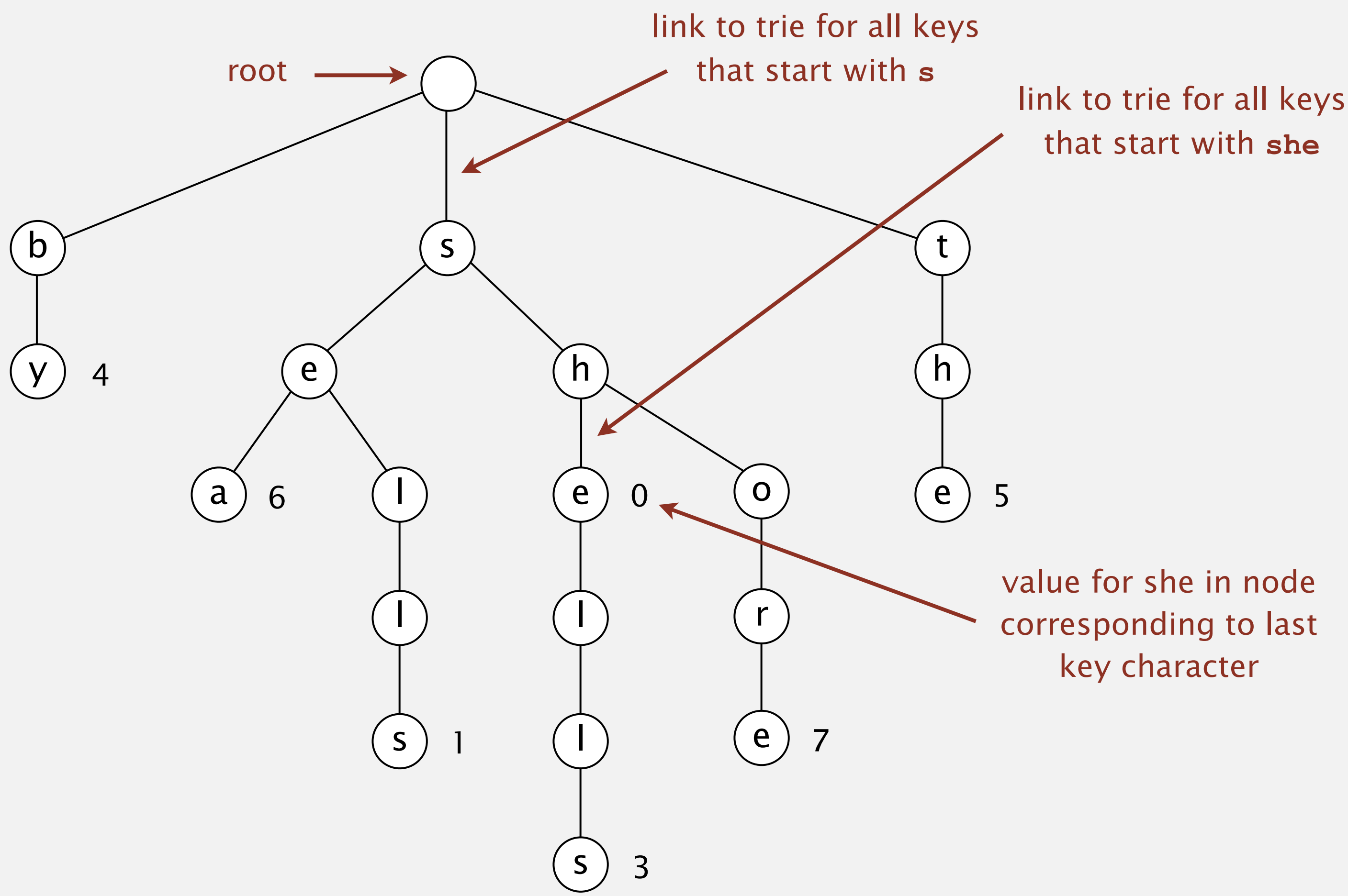
- R-way tries
- ternary search tries
- character-based operations

Tries

Tries. [from retrieval, but pronounced "try"]

- Store characters in nodes (not keys).
- Each node has R children, one for each possible character.
- For now, we do not draw null links.

key	value
by	4
sea	6
sells	1
she	0
shells	3
shore	7
the	5

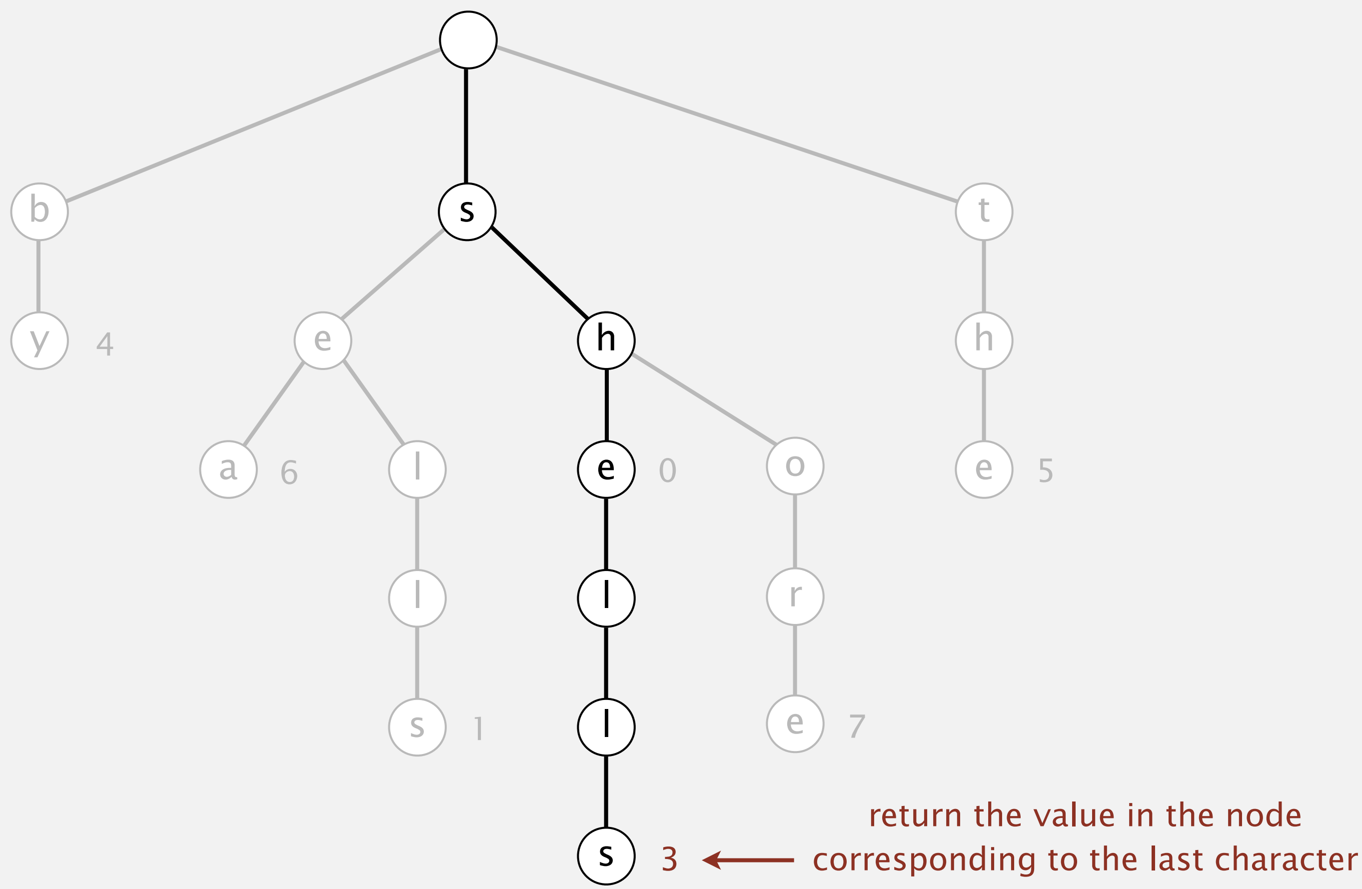


Search in a trie

Follow links corresponding to each character in the key.

- Search hit: node where search ends has a non-null value.
- Search miss: reach a null link or node where search ends has null value.

get("shells")



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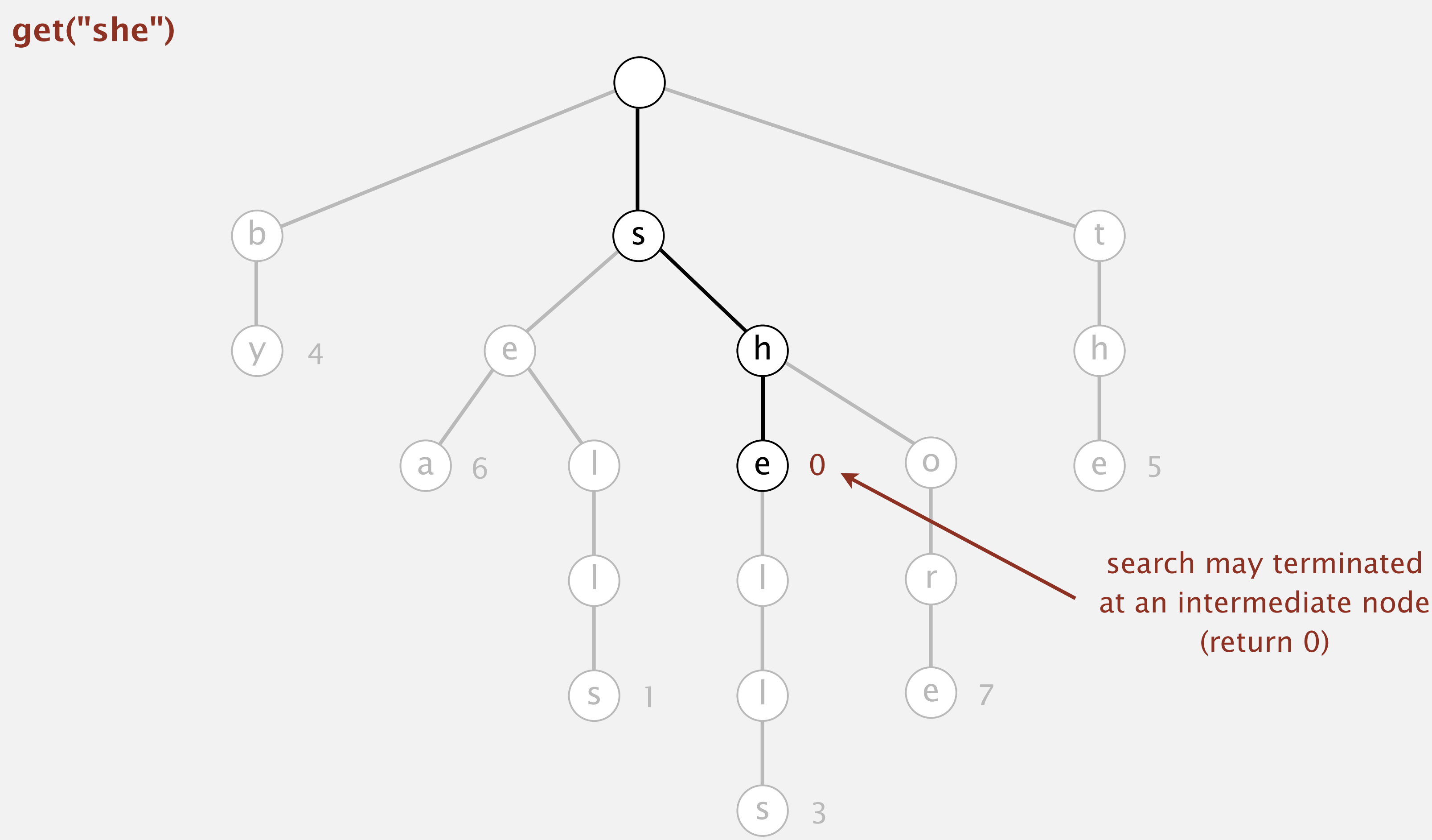
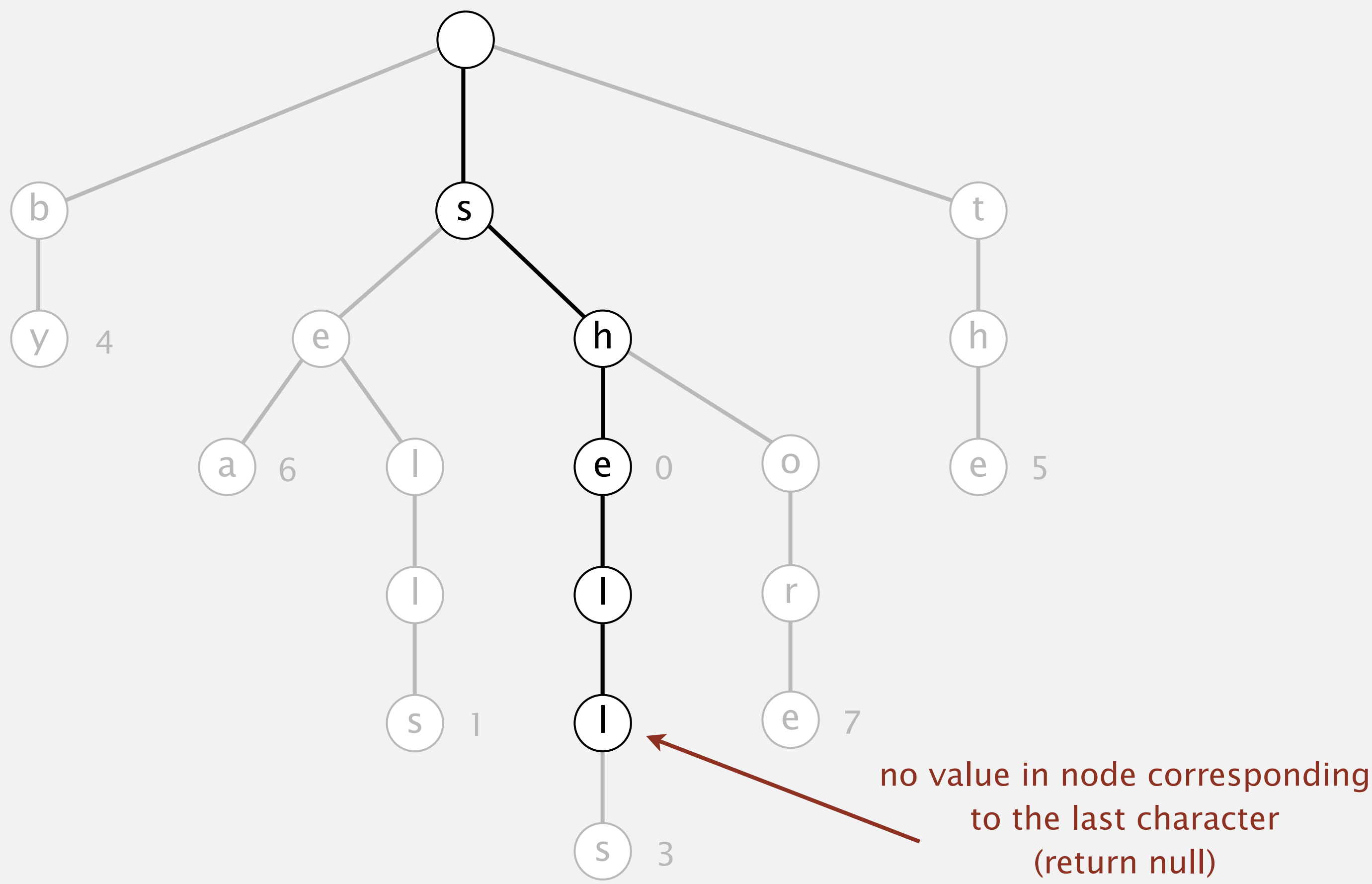


Search in a trie

Follow links corresponding to each character in the key.

- Search hit: node where search ends has a non-null value.
- Search miss: reach a null link or node where search ends has null value.

get("shell")



Search in a trie

Follow links corresponding to each character in the key.

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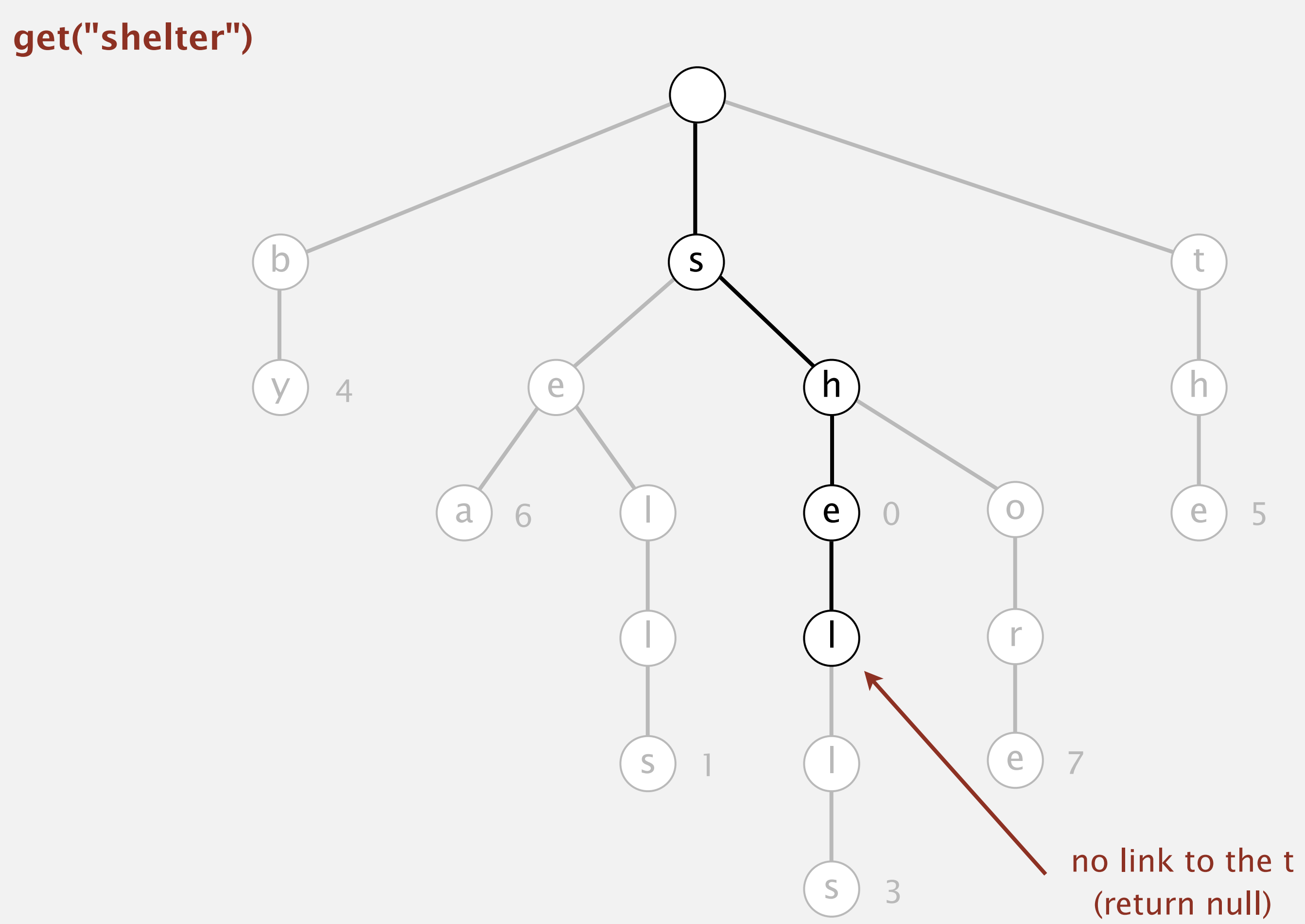
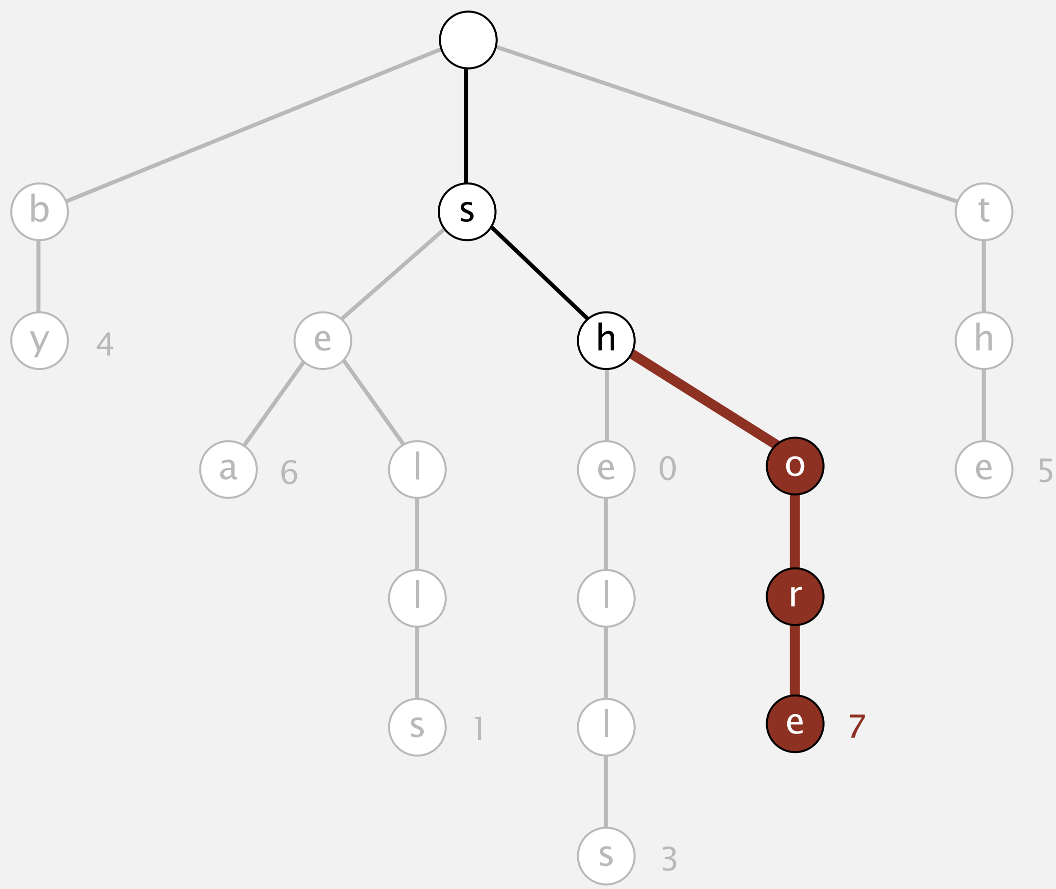


Insertion into a trie

Follow links corresponding to each character in the key.

- Encounter a null link: create new node.
- Encounter the last character of the key: set value in that node.

put("shore", 7)

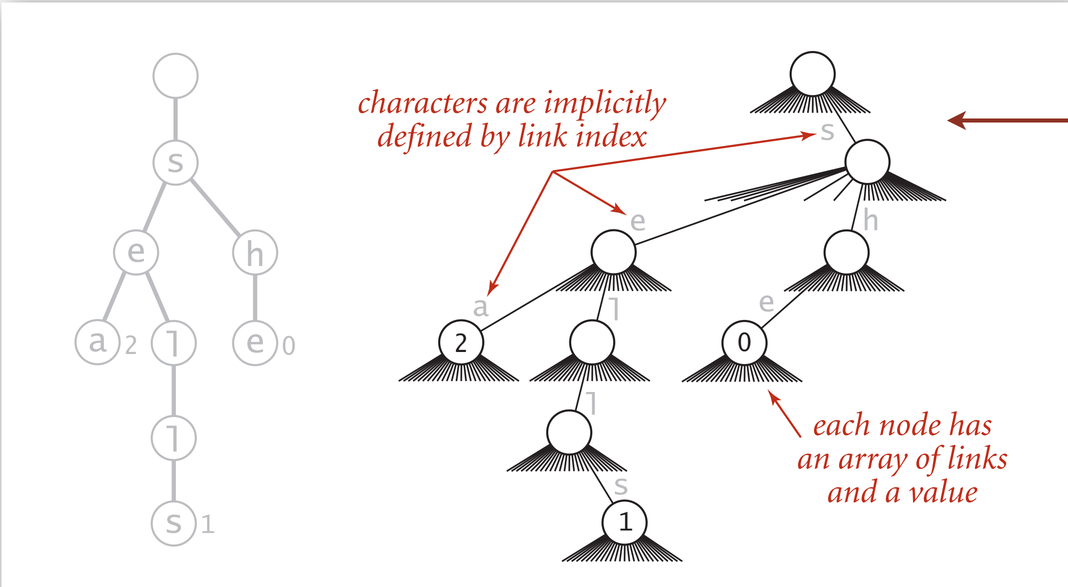


Trie construction demo

Node. A value, plus references to R nodes.

```
private static class Node
{
   private Object value;
   private Node[] next = new Node[R];
}
```

use **Object** instead of **Value** since no generic array creation in Java



Trie representation

keys are not explicitly stored

R-way trie: Java implementation

```
public class TrieST<Value>
{
   private static final int R = 256;
   private Node root;

   private static class Node
   {  /* see previous slide */  }

   public void put(String key, Value val)
   {  root = put(root, key, val, 0);  }

   private Node put(Node x, String key, Value val, int d)
   {
      if (x == null) x = new Node();
      if (d == key.length()) { x.val = val; return x; }
      char c = key.charAt(d);
      x.next[c] = put(x.next[c], key, val, d+1);
      return x;
   }

}
```

extended ASCII

R-way trie: Java implementation (continued)

```
public boolean contains(String key)
{  return get(key) != null;  }

public Value get(String key)
{
   Node x = get(root, key, 0);
   if (x == null) return null;
   return (Value) x.val;
}
```

cast needed

```
private Node get(Node x, String key, int d)
{
   if (x == null) return null;
   if (d == key.length()) return x;
   char c = key.charAt(d);
   return get(x.next[c], key, d+1);
}
```

Trie performance

Search miss.

- Could have mismatch on first character.
- Typical case: examine only a few characters (sublinear).

Search hit. Need to examine all L characters for equality.

Space. R null links at each leaf.

(but sublinear space possible if many short strings share common prefixes)

Bottom line. Fast search hit and even faster search miss, but wastes space.

String symbol table implementations cost summary

	character accesses (typical case)				dedup	
implementation	search hit	search miss	insert	space (references)	moby.txt	actors.txt
red-black BST	$L + c \lg^2 N$	$c \lg^2 N$	$c \lg^2 N$	4N	1.40	97.4
hashing	L	L	L	4N to 16N	0.76	40.6
R-way trie	L	$\log_R N$	L	(R+1) N	1.12	out of memory

R-way trie.

- Method of choice for small R.
- Too much memory for large R.

Challenge. Use less memory, e.g., 65,536-way trie for Unicode!

Digression: out of memory?

" 640 K ought to be enough for anybody. "

— attributed to Bill Gates, 1981

(commenting on the amount of RAM in personal computers)

" 64 MB of RAM may limit performance of some Windows XP features; therefore, 128 MB or higher is recommended for best performance. " *— Windows XP manual, 2002*

" 64 bit is coming to desktops, there is no doubt about that. But apart from Photoshop, I can't think of desktop applications where you would need more than 4GB of physical memory, which is what you have to have in order to benefit from this technology. Right now, it is costly. " *— Bill Gates, 2003*

Digression: out of memory?

A short (approximate) history.

machine	year	address bits	addressable memory	typical actual memory	cost
PDP-8	1960s	12	6 KB	6 KB	$16K
PDP-10	1970s	18	256 KB	256 KB	$1M
IBM S/360	1970s	24	4 MB	512 KB	$1M
VAX	1980s	32	4 GB	1 MB	$1M
Pentium	1990s	32	4 GB	1 GB	$1K
Xeon	2000s	64	enough	4 GB	$100
??	future	128+	enough	enough	$1

" 512-bit words ought to be enough for anybody. "
— Kevin Wayne, 1995

A modest proposal

Number of atoms in the universe (estimated). $\leq 2^{266}$.

Age of universe (estimated). 14 billion years ~ 2^{59} seconds $\leq 2^{89}$ nanoseconds.

Q. How many bits address every atom that ever existed?

A. Use a unique 512-bit address for every atom at every time quantum.

266 bits	89 bits	157 bits
atom	time	cushion for whatever

Ex. Use 256-way trie to map each atom to location.

- Represent atom as 64 8-bit chars (512 bits).
- 256-way trie wastes 255/256 actual memory.
- Need better use of memory.

- R-way tries
- **ternary search tries**
- character-based operations

- Store characters and values in nodes (not keys).
- Each node has three children: smaller (left), equal (middle), larger (right).

Fast Algorithms for Sorting and Searching Strings

Jon L. Bentley*   Robert Sedgewick#

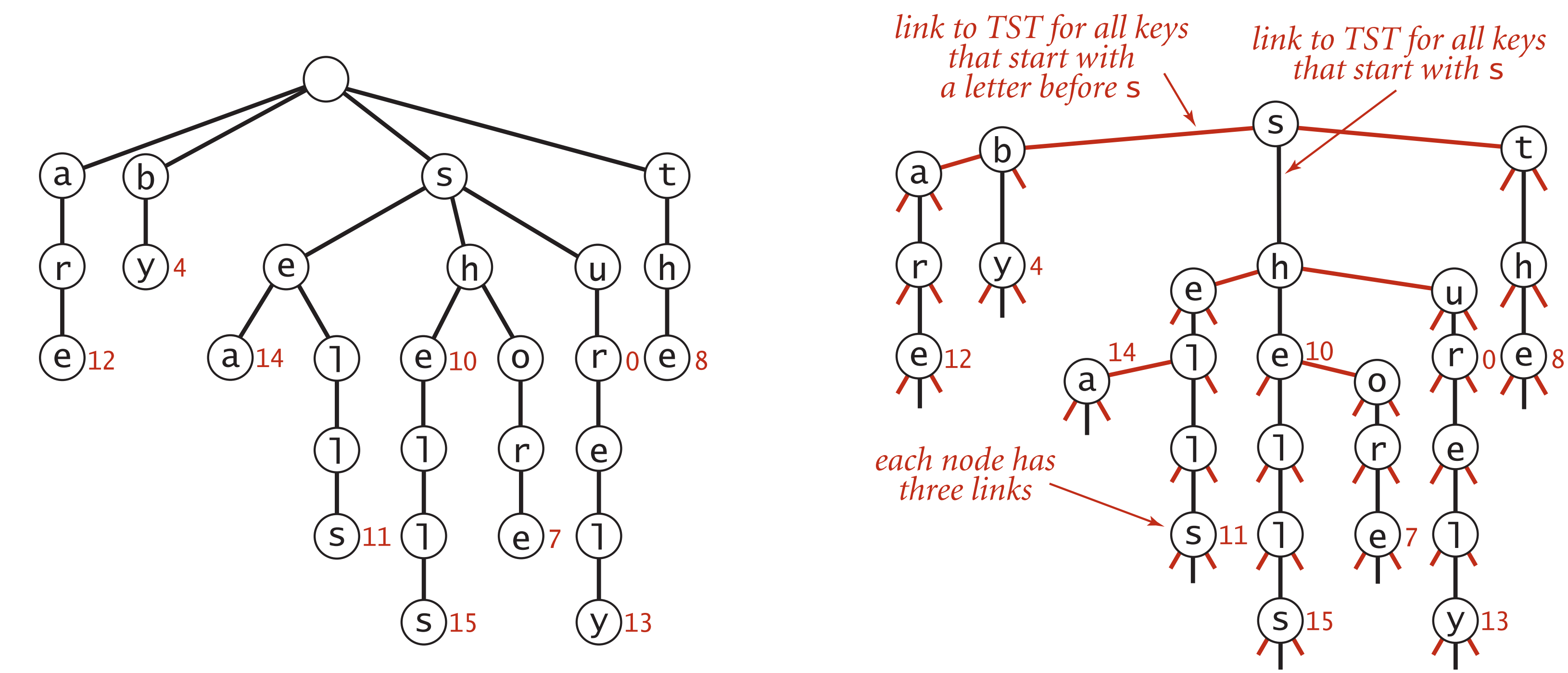
Abstract

We present theoretical algorithms for sorting and searching multikey data, and derive from them practical C implementations for applications in which keys are character strings. The sorting algorithm blends Quicksort and radix sort; it is competitive with the best known C sort codes. The searching algorithm blends tries and binary search trees; it is faster than hashing and other commonly used search methods. The basic ideas behind the algo- that is competitive with the most efficient string sorting programs known. The second program is a symbol table implementation that is faster than hashing, which is commonly regarded as the fastest symbol table implementation. The symbol table implementation is much more space-efficient than multiway trees, and supports more advanced searches.

In many application programs, sorts use a Quicksort implementation based on an abstract compare operation,

Ternary search tries

- Store characters and values in nodes (not keys).
- Each node has three children: smaller (left), equal (middle), larger (right).



TST representation of a trie

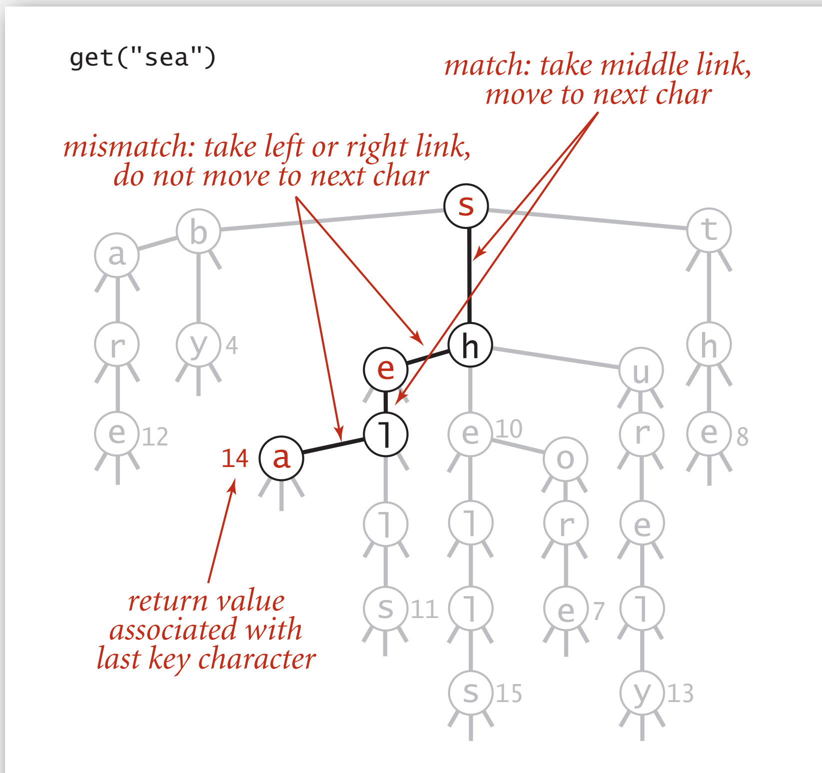
Search in a TST

Follow links corresponding to each character in the key.

- If less, take left link; if greater, take right link.
- If equal, take the middle link and move to the next key character.

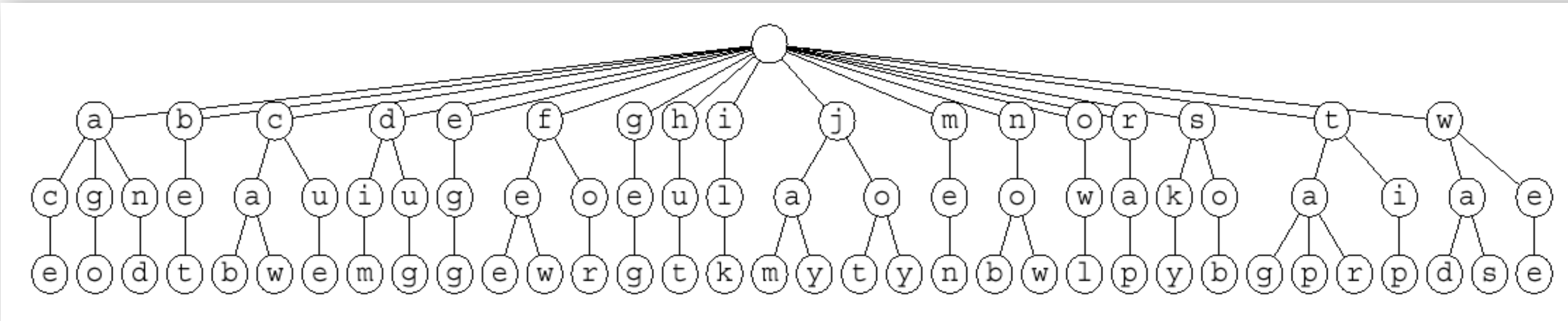
Search hit. Node where search ends has a non-null value.

Search miss. Reach a null link or node where search ends has null value.



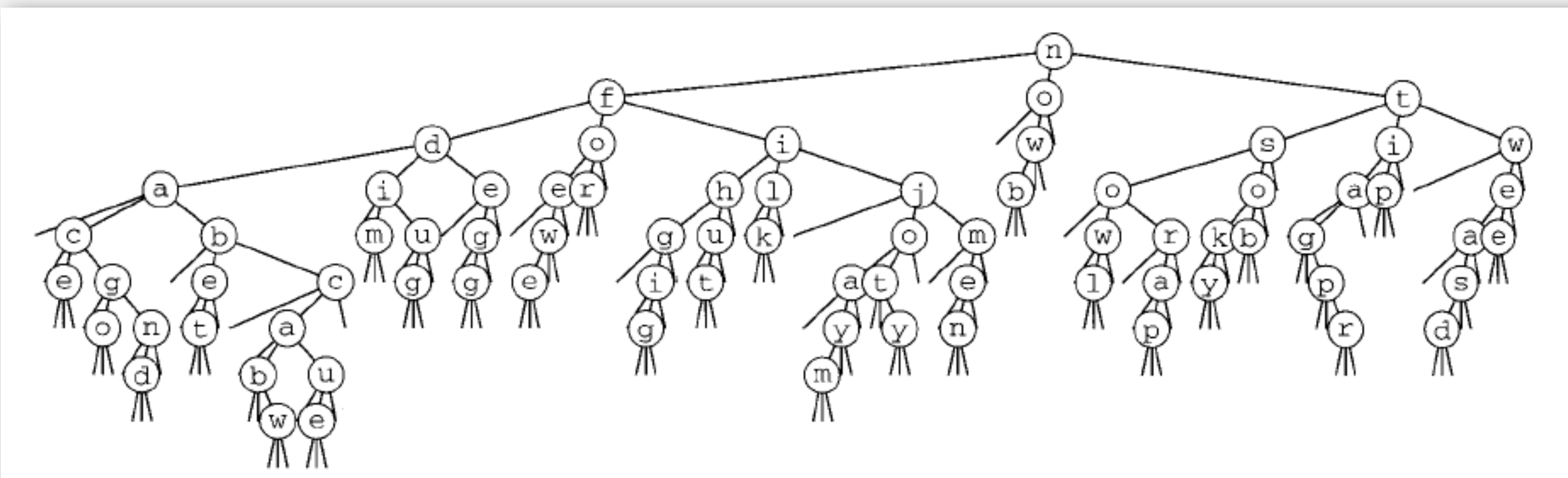
26-way trie vs. TST

26-way trie. 26 null links in each leaf.



26-way trie (1035 null links, not shown)

TST. 3 null links in each leaf.



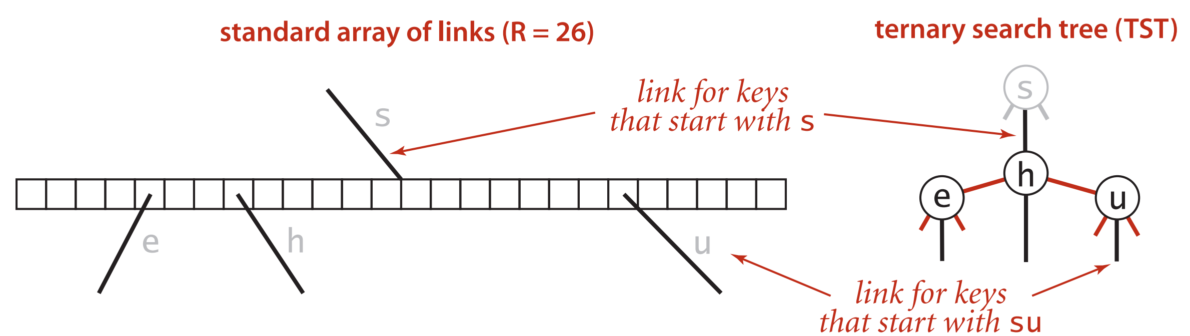
TST (155 null links)

```
now
for
tip
ilk
dim
tag
jot
sob
nob
sky
hut
ace
bet
men
egg
few
jay
owl
joy
rap
gig
wee
was
cab
wad
caw
cue
fee
tap
ago
tar
jam
dug
and
```

A TST node is five fields:

- A value.
- A character c.
- A reference to a left TST.
- A reference to a middle TST.
- A reference to a right TST.

```
private class Node
{
   private Value val;
   private char c;
   private Node left, mid, right;
}
```



Trie node representations

TST: Java implementation

```
public class TST<Value>
{
   private Node root;

   private class Node
   {  /* see previous slide */  }

   public void put(String key, Value val)
   {  root = put(root, key, val, 0);  }

   private Node put(Node x, String key, Value val, int d)
   {
      char c = key.charAt(d);
      if (x == null) {  x = new Node(); x.c = c;  }
      if      (c < x.c)            x.left  = put(x.left,  key, val, d);
      else if (c > x.c)            x.right = put(x.right, key, val, d);
      else if (d < s.length() - 1) x.mid   = put(x.mid,   key, val, d+1);
      else                         x.val   = val;
      return x;
   }

}
```

TST: Java implementation (continued)

```
public boolean contains(String key)
{  return get(key) != null;  }

public Value get(String key)
{
   Node x = get(root, key, 0);
   if (x == null) return null;
   return x.val;
}

private Node get(Node x, String key, int d)
{
   if (x == null) return null;
   char c = key.charAt(d);
   if      (c < x.c)              return get(x.left,  key, d);
   else if (c > x.c)              return get(x.right, key, d);
   else if (d < key.length() - 1) return get(x.mid,   key, d+1);
   else                           return x;
}
```

String symbol table implementation cost summary

	character accesses (typical case)				dedup	
implementation	search hit	search miss	insert	space (references)	moby.txt	actors.txt
red-black BST	$L + c \lg^2 N$	$c \lg^2 N$	$c \lg^2 N$	$4N$	1.40	97.4
hashing	L	L	L	$4N$ to $16N$	0.76	40.6
R-way trie	L	$\log_R N$	L	$(R+1)N$	1.12	out of memory
TST	$L + \ln N$	$\ln N$	$L + \ln N$	$4N$	0.72	38.7

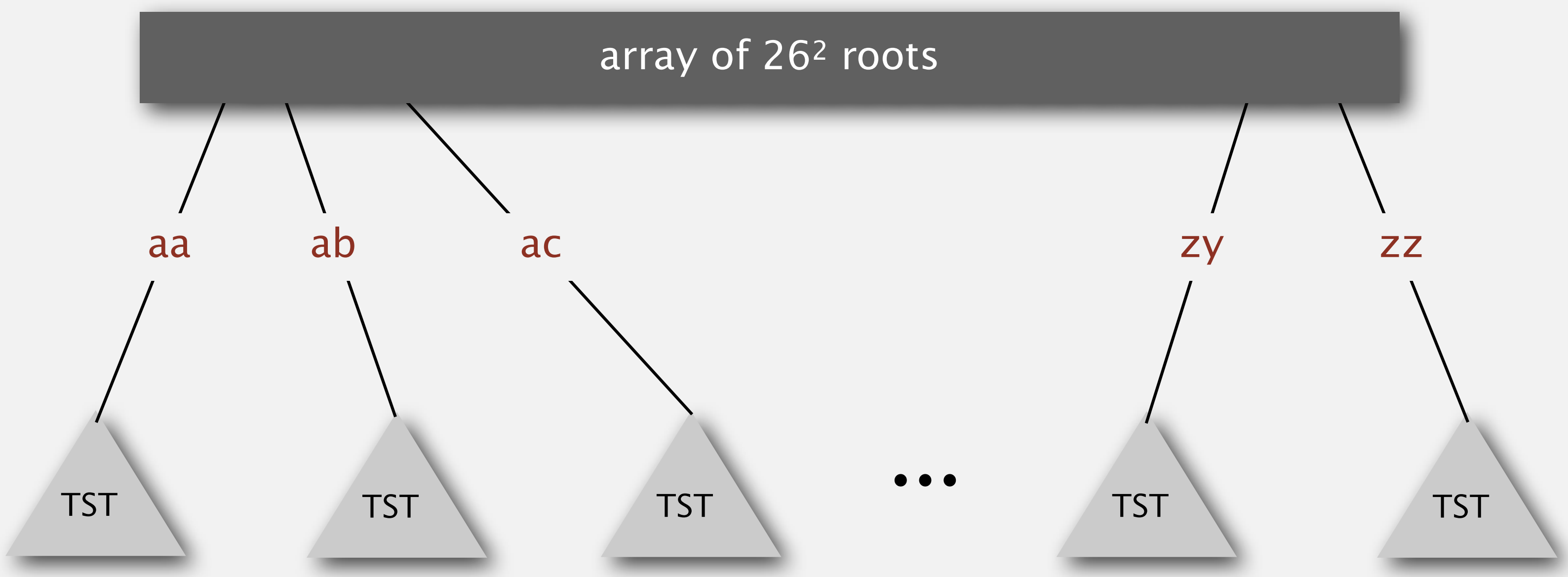
Remark. Can build balanced TSTs via rotations to achieve $L + \log N$ worst-case guarantees.

Bottom line. TST is as fast as hashing (for string keys), space efficient.

TST with R^2 branching at root

Hybrid of R-way trie and TST.

- Do R^2-way branching at root.
- Each of R^2 root nodes points to a TST.



Q. What about one- and two-letter words?

String symbol table implementation cost summary

	character accesses (typical case)				dedup	
implementation	search hit	search miss	insert	space (references)	moby.txt	actors.txt
red-black BST	$L + c \lg^2 N$	$c \lg^2 N$	$c \lg^2 N$	$4N$	1.40	97.4
hashing	L	L	L	$4N$ to $16N$	0.76	40.6
R-way trie	L	$\log_R N$	L	$(R + 1) N$	1.12	out of memory
TST	$L + \ln N$	$\ln N$	$L + \ln N$	$4N$	0.72	38.7
TST with R^2	$L + \ln N$	$\ln N$	$L + \ln N$	$4N + R^2$	0.51	32.7

TST vs. hashing

Hashing.

- Need to examine entire key.
- Search hits and misses cost about the same.
- Need good hash function for every key type.
- Does not support ordered symbol table operations.

TSTs.

- Works only for strings (or digital keys).
- Only examines just enough key characters.
- Search miss may only involve a few characters.
- Supports ordered symbol table operations (plus others!).

Bottom line. TSTs are:

- Faster than hashing (especially for search misses).
  More flexible than red-black BSTs. [stay tuned]

- R-way tries
- ternary search tries
- **character-based operations**

String symbol table API

Character-based operations. The string symbol table API supports several useful character-based operations.

key	value
by	4
sea	6
sells	1
she	0
shells	3
shore	7
the	5

Prefix match. Keys with prefix `"sh"`: `"she"`, `"shells"`, and `"shore"`.

Wildcard match. Keys that match `".he"`: `"she"` and `"the"`.

Longest prefix. Key that is the longest prefix of `"shellsort"`: `"shells"`.

String symbol table API

`public class StringST<Value>`

	`StringST()`	*create a symbol table with string keys*
`void`	`put(String key, Value val)`	*put key-value pair into the symbol table*
`Value`	`get(String key)`	*value paired with key*
`void`	`delete(String key)`	*delete key and corresponding value*
	⋮	
`Iterable<String>`	`keys()`	*all keys*
`Iterable<String>`	`keysWithPrefix(String s)`	*keys having s as a prefix*
`Iterable<String>`	`keysThatMatch(String s)`	*keys that match s (where . is a wildcard)*
`String`	`longestPrefixOf(String s)`	*longest key that is a prefix of s*

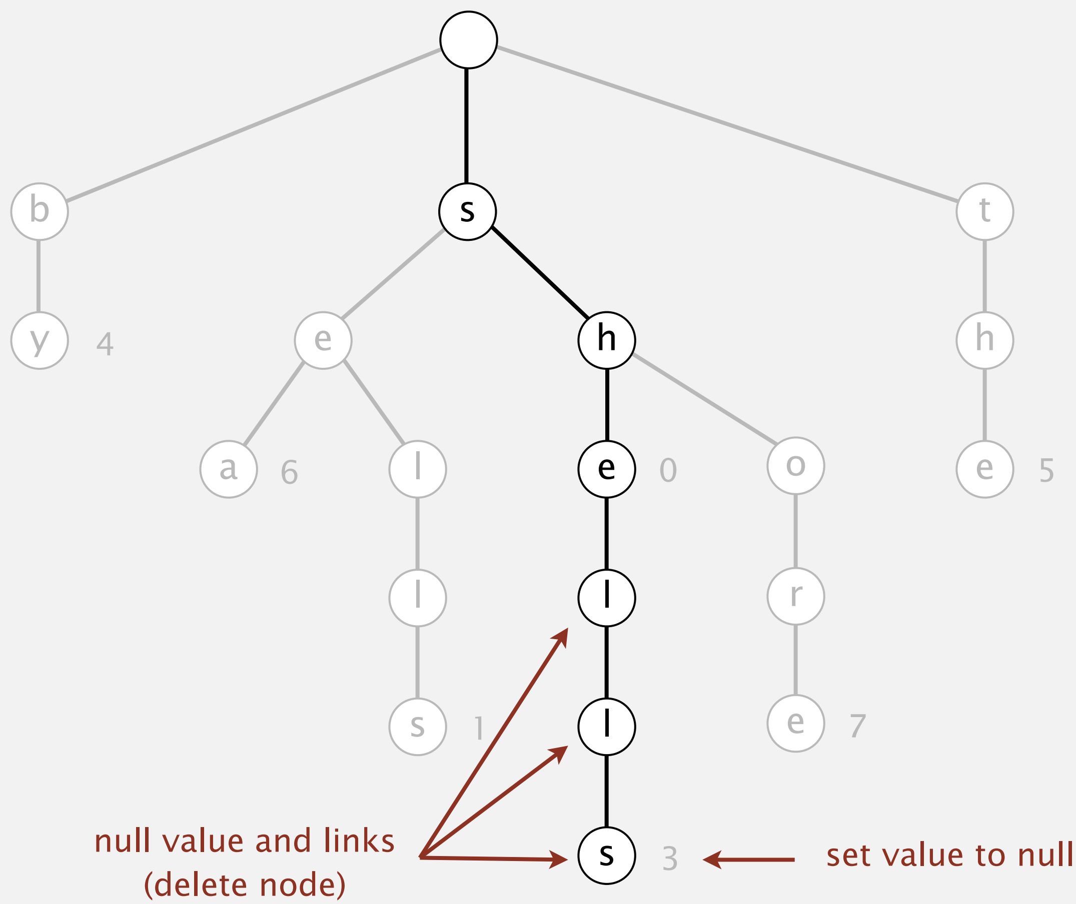
Remark. Can also add other ordered ST methods, e.g., `floor()` and `rank()`.

Deletion in an R-way trie

To delete a key-value pair:

- Find the node corresponding to key and set value to null.
- If that node has all null links, remove that node (and recur).

delete("shells")



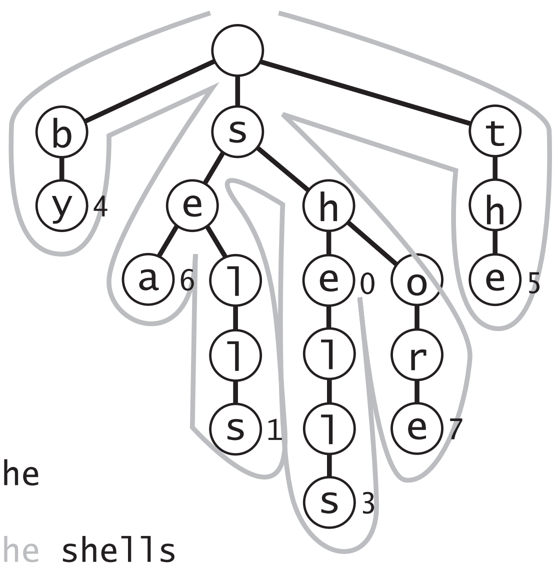
Ordered iteration

To iterate through all keys in sorted order:

- Do inorder traversal of trie; add keys encountered to a queue.
- Maintain sequence of characters on path from root to node.

```
keysWithPrefix("");
```

key	q
b	
by	by
s	
se	
sea	by sea
sel	
sell	
sells	by sea sells
sh	
she	by sea sells she
shell	
shells	by sea sells she shells
sho	
shor	
shore	by sea sells she shells shore
t	
th	
the	by sea sells she shells shore the



Ordered iteration: Java implementation

To iterate through all keys in sorted order:

- Do inorder traversal of trie; add keys encountered to a queue.
- Maintain sequence of characters on path from root to node.

```
public Iterable<String> keys()
{
   Queue<String> queue = new Queue<String>();
   collect(root, "", queue);
   return queue;
}

private void collect(Node x, String prefix, Queue<String> q)
{
   if (x == null) return;
   if (x.val != null) q.enqueue(prefix);
   for (char c = 0; c < R; c++)
      collect(x.next[c], prefix + c, q);
}
```

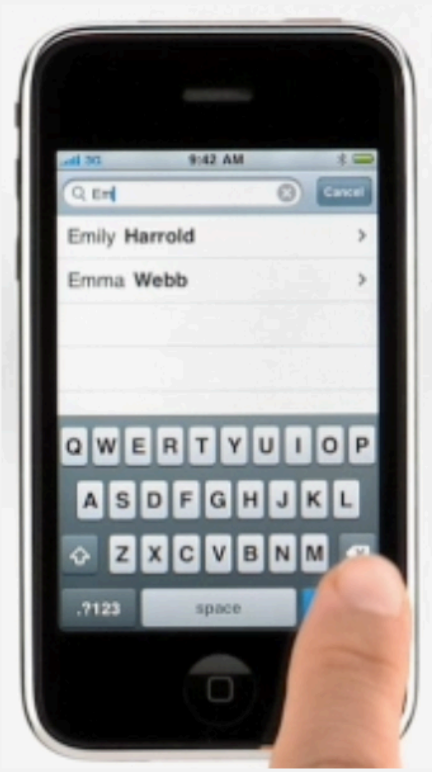
sequence of characters on path from root to x

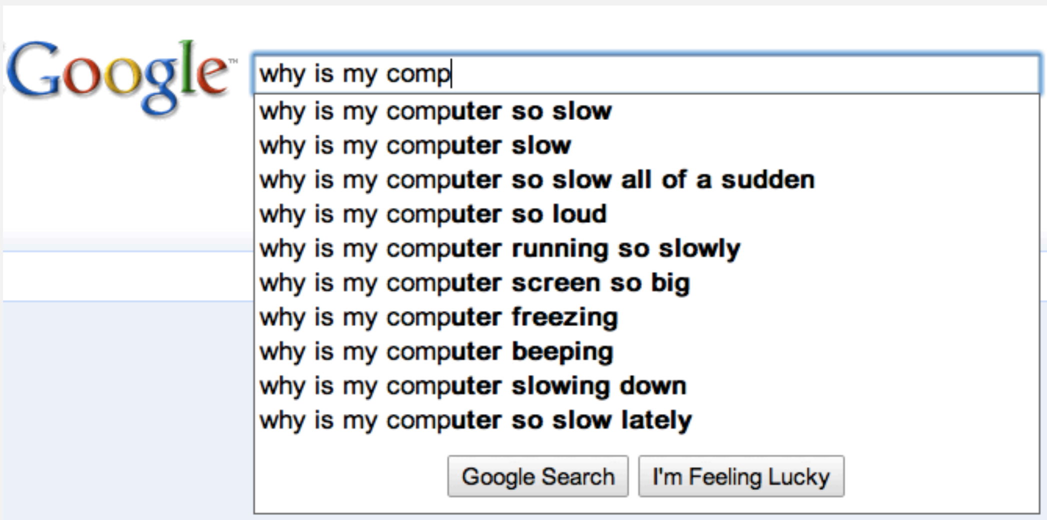
Prefix matches

Find all keys in symbol table starting with a given prefix.

Ex. Autocomplete in a cell phone, search bar, text editor, or shell.

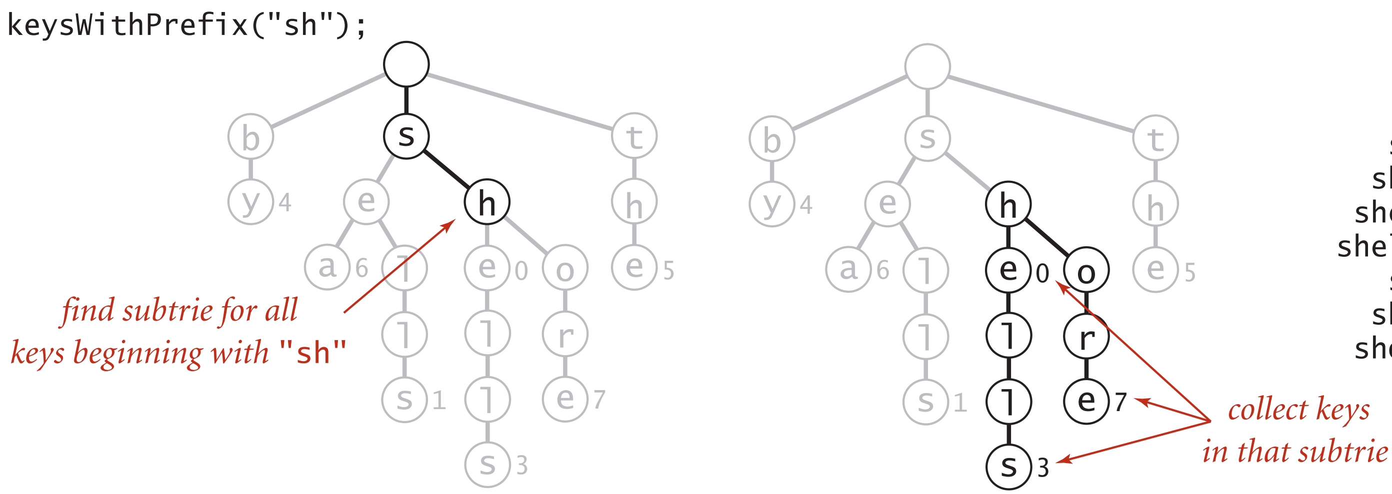
- User types characters one at a time.
- System reports all matching strings.





Prefix matches

Find all keys in symbol table starting with a given prefix.



key	q
sh	
she	she
shel	
shell	
shells	she shells
sho	
shor	
shore	she shells shore

Prefix match in a trie

```
public Iterable<String> keysWithPrefix(String prefix)
{
   Queue<String> queue = new Queue<String>();
   Node x = get(root, prefix, 0);
   collect(x, prefix, queue);
   return queue;
}
```

root of subtrie for all strings beginning with given prefix

Longest prefix

Find longest key in symbol table that is a prefix of query string.

Ex. To send packet toward destination IP address, router chooses IP address in routing table that is longest prefix match.

```
"128"
"128.112"
"128.112.055"
"128.112.055.15"
"128.112.136"
"128.112.155.11"
"128.112.155.13"
"128.222"
"128.222.136"
```

represented as 32-bit binary number for IPv4 (instead of string)

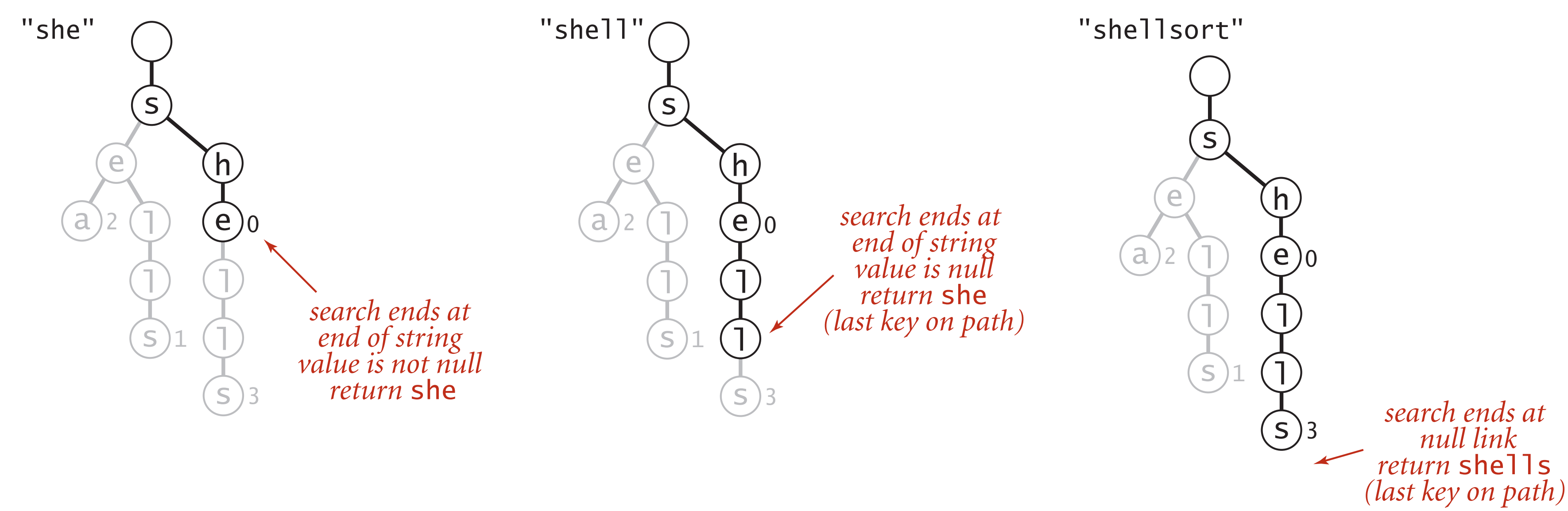
```
longestPrefixOf("128.112.136.11") = "128.112.136"
longestPrefixOf("128.112.100.16") = "128.112"
longestPrefixOf("128.166.123.45") = "128"
```

Note. Not the same as floor: `floor("128.112.100.16") = "128.112.055.15"`

Longest prefix

Find longest key in symbol table that is a prefix of query string.

- Search for query string.
- Keep track of longest key encountered.



Possibilities for `longestPrefixOf()`

Longest prefix: Java implementation

Find longest key in symbol table that is a prefix of query string.

- Search for query string.
- Keep track of longest key encountered.

```
public String longestPrefixOf(String query)
{
   int length = search(root, query, 0, 0);
   return query.substring(0, length);
}

private int search(Node x, String query, int d, int length)
{
   if (x == null) return length;
   if (x.val != null) length = d;
   if (d == query.length()) return length;
   char c = query.charAt(d);
   return search(x.next[c], query, d+1, length);
}
```

T9 texting

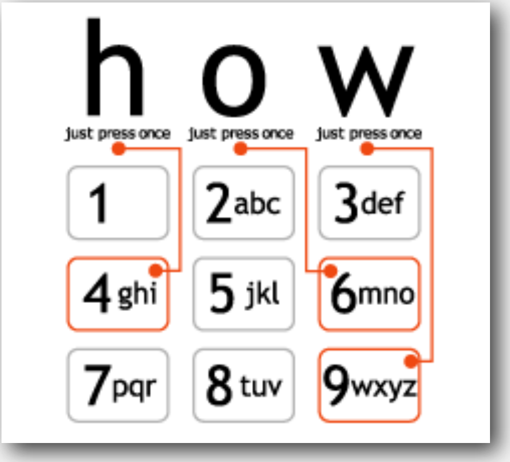
Goal. Type text messages on a phone keypad.

Multi-tap input. Enter a letter by repeatedly pressing a key until the desired letter appears.

"a much faster and more fun way to enter text"

T9 text input.

- Find all words that correspond to given sequence of numbers.
- Press 0 to see all completion options.



www.t9.com

Ex. **hello**

- Multi-tap: `4 4 3 3 5 5 5 5 5 5 6 6 6`
- T9: `4 3 5 5 6`

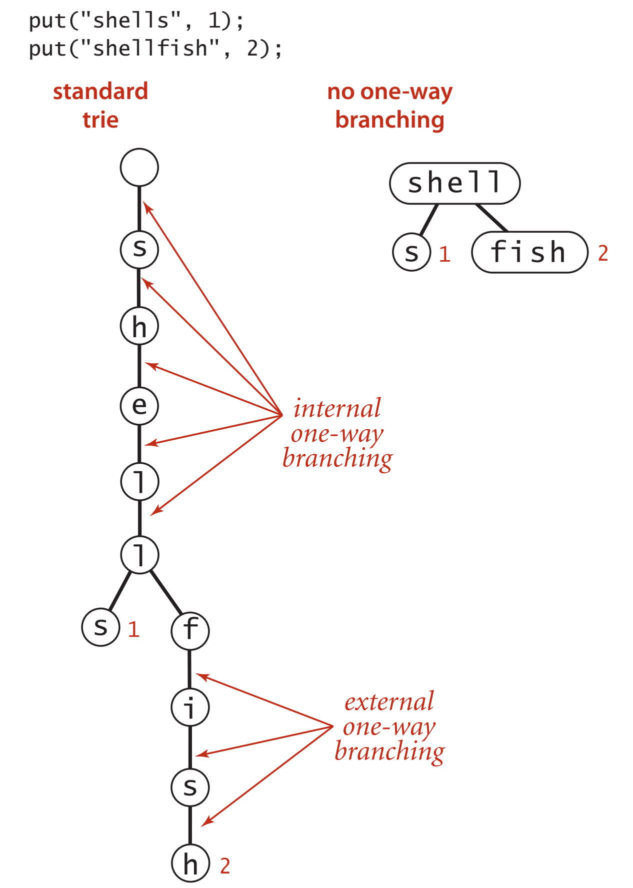
Compressing a trie

Collapsing 1-way branches at bottom.

Internal node stores character; leaf node stores suffix (or full key).

Collapsing interior 1-way branches.

Node stores a sequence of characters.

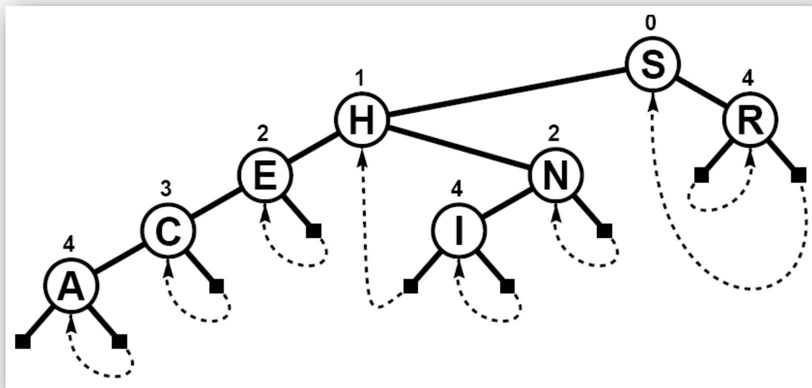


Removing one-way branching in a trie

A classic algorithm

Patricia tries. [Practical Algorithm to Retrieve Information Coded in Alphanumeric]

- Collapse one-way branches in binary trie.
- Thread trie to eliminate multiple node types.



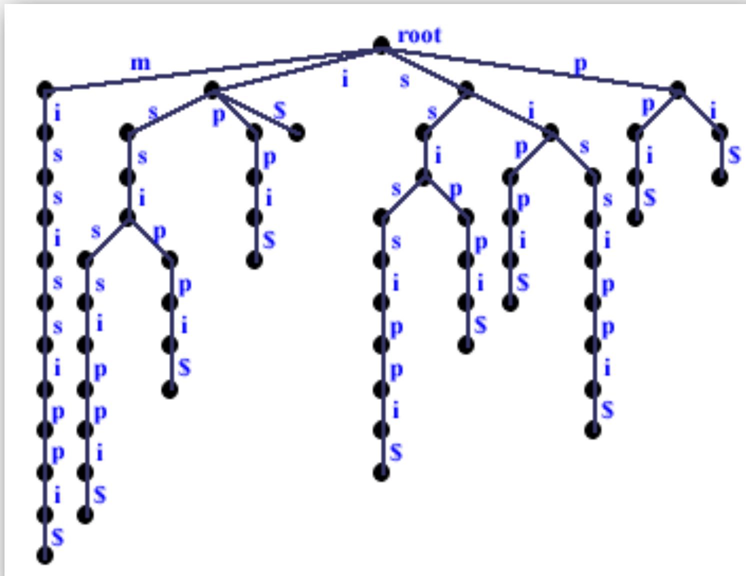
Applications.

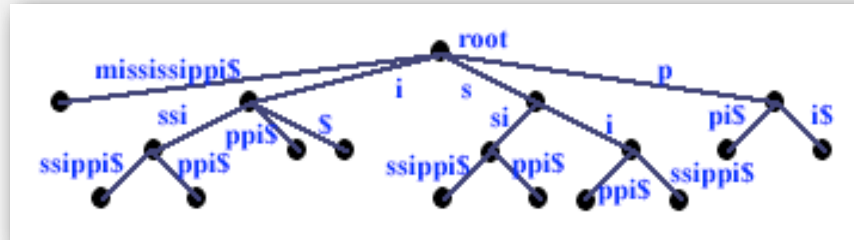
- Database search.
- P2P network search.
- IP routing tables: find longest prefix match.
- Compressed quad-tree for N-body simulation.
- Efficiently storing and querying XML documents.

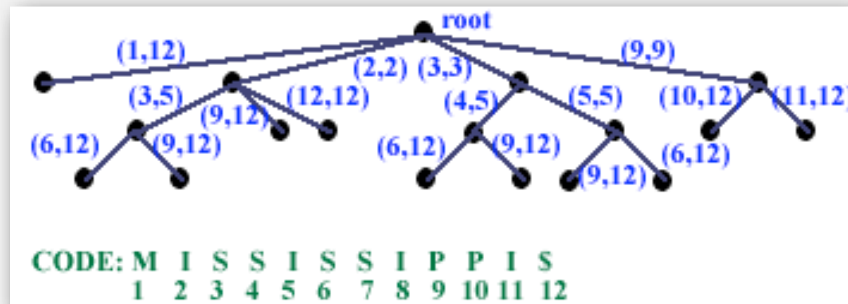
Implementation. One step beyond this lecture.

Suffix tree

Suffix tree. Threaded trie with collapsed 1-way branching for string suffixes.







Applications.
- Linear-time longest repeated substring.
- Computational biology databases (BLAST, FASTA).

Implementation. One step beyond this lecture.

String symbol tables summary

A success story in algorithm design and analysis.

Red-black BST.

- Performance guarantee: $\log N$ key compares.
- Supports ordered symbol table API.

Hash tables.

- Performance guarantee: constant number of probes.
- Requires good hash function for key type.

Tries. R-way, TST.

- Performance guarantee: $\log N$ characters accessed.
- Supports character-based operations.

Bottom line. You can get at anything by examining 50-100 bits (!!!)