(1)

Define a function GRC( ) as follows:

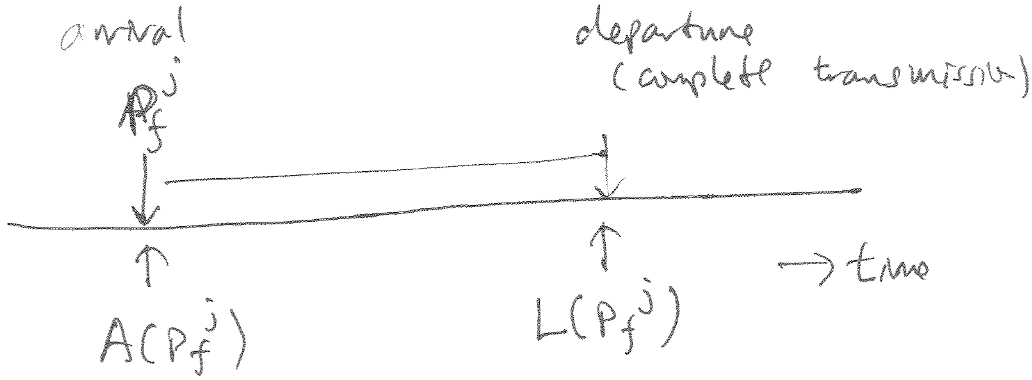
$$GRC(P_f^{\emptyset}) = \emptyset \qquad \text{(initial value)}$$

$$GRC(P_f^{j}) = \max\{A(P_f^j), GRC(P_f^{j-1})\} + \frac{l_f^j}{r_f}$$

$A(P_f^j)$ = arrival time of packet P_f^j

l_f^j = length (# bytes) of packet P_f^j.

r_f = reserved output data rate for flow f.



<u>Magic</u>: A scheduling algorithm belongs to GR (Guarantee Rate) if the scheduling alg. guarantees that packet P_f^j will be transmitted (completely) ~~by~~ at time t

$$t \leq GRC(P_f^j) + \beta$$

for some constant β for all packet P_f^j

Claim: FFS is a GR scheduler.

(a) Let $L_{GPS}(p_f^j)$ = completion time of packet p_f^j under GPS.

(b) $L_{GPS}(p_f^0) = \phi$.

(c) Let each flow reserve a rate $r_1, r_2 \ldots r_m$

Total capacity C.

Then flow f receives a service rate of

$$\frac{r_f}{r_1 + r_2 + \cdots + r_m} \cdot C$$

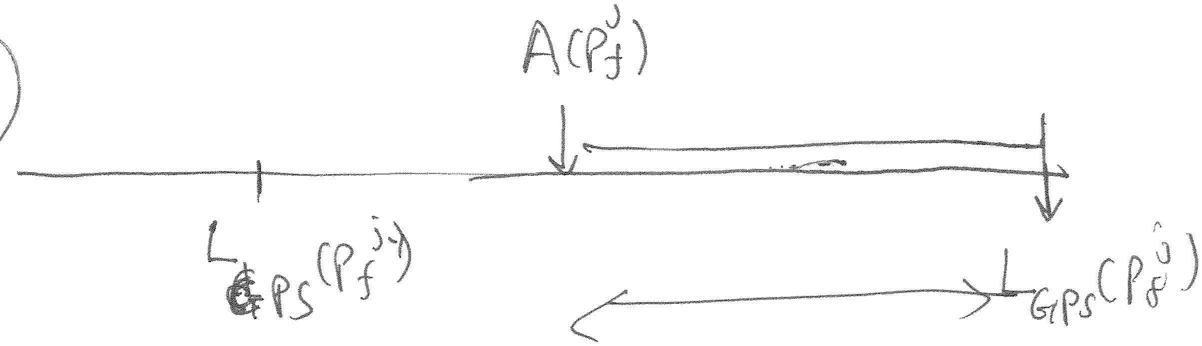
$$= \frac{C}{\underbrace{r_1 + r_2 + \cdots + r_m}_{\leq C}} \cdot r_f$$

$$\geq r_f .$$

(d) The packet p_f^j of length l_f^j will complete transmission in $\leq \frac{l_f^j}{r_f}$ seconds.

(e) Question: when will P_f^j finish if P_f^{j-1} finish at $L_{GPS}(P_f^j)$.

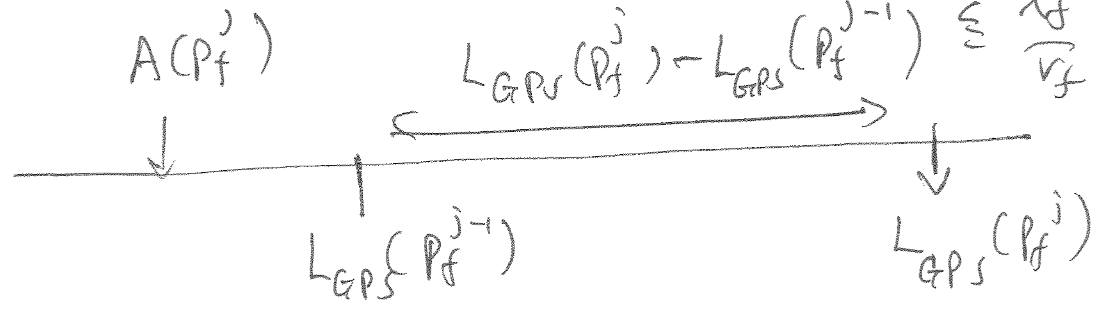
1



$$L_{GPS}(P_f^j) - A(P_f^j) \leq \frac{l_f^j}{r_f}.$$

2

$$L_{GPS}(P_f^j) - L_{GPS}(P_f^{j-1}) \leq \frac{l_f^j}{r_f}$$



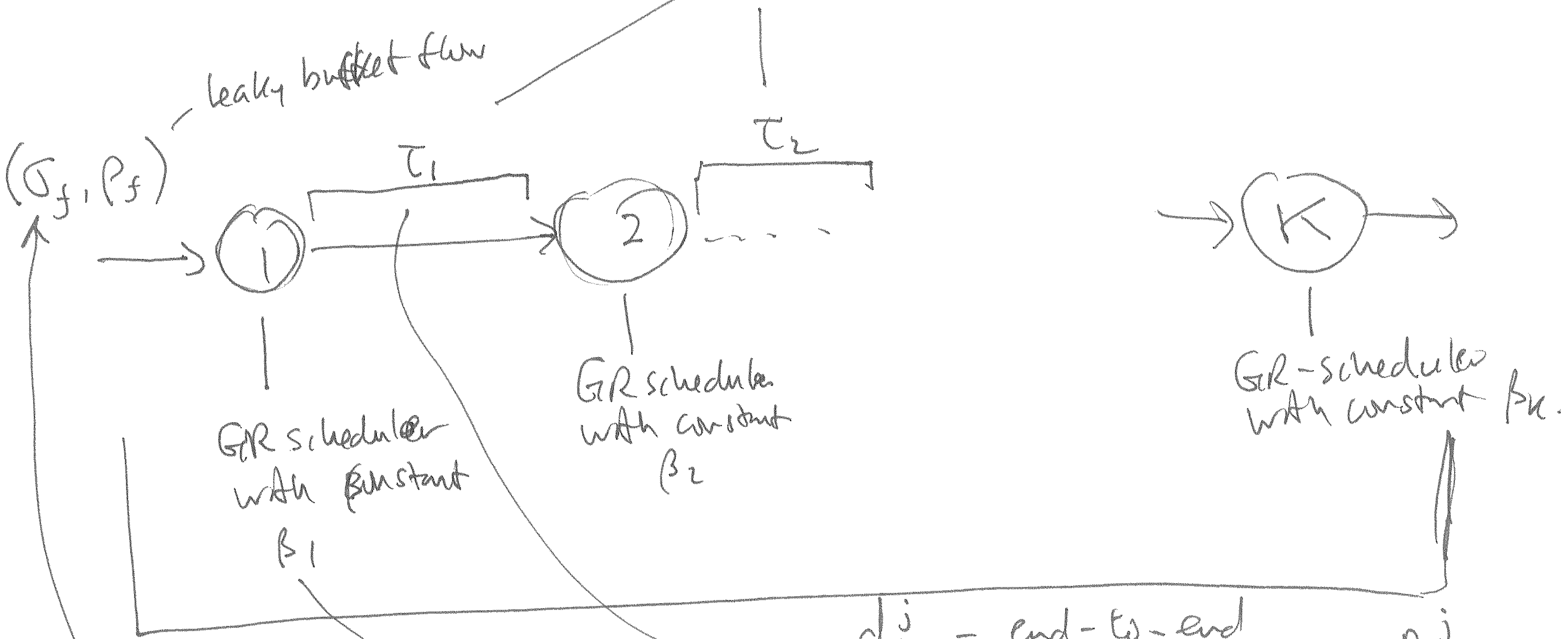
Conclusion:

$$\underline{L_{GPS}(P_f^j)} \leq \max\left(A(P_f^j), L_{GPS}(P_f^{j-1})\right) + \frac{l_f^j}{r_f}.$$

$\Rightarrow \beta = \phi$ (perfect fairness).

Theorem :

propagation delay



d_f^j = end-to-end delay of packet P_f^j

$$d_f^j \leq \frac{\sigma_f + (K-1)\cdot M}{r_f} + \sum_{i=1}^{K} (\beta_i + \tau_i)$$

Max. packet size